

**Introducing
Dynamic Mathematics Software
to Mathematics Teachers:
the Case of GeoGebra**

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Chapter 1

Introduction

1.1 Abstract

This dissertation aims to identify effective approaches for introducing dynamic mathematics software to secondary school mathematics teachers, and to develop corresponding instructional materials for professional development in the use of this software and technology. Based upon an analysis of introductory workshops for the dynamic mathematics software GeoGebra,

- frequently occurring difficulties and impediments that arise during the introduction process as well as challenging tools and features of GeoGebra were identified in this study,
- a set of complexity criteria for dynamic geometry tools were established that permit the classification of such tools according to their general difficulty level in order to facilitate their introduction to novices, and
- workshops and accompanying instructional materials are being designed in order to reduce common impediments that arise during the introduction process of dynamic mathematics software, and to enable teachers to more effectively integrate GeoGebra into their teaching practices.

The instructional materials derived from the analysis of identified impediments will provide a basis for future professional development with GeoGebra offered by the International GeoGebra Institute with the goal of supporting mathematics teachers who would like to effectively integrate dynamic mathematics software into their teaching practices.

1.2 Objectives of the Thesis

In the context of this dissertation, literature about technology integration into mathematics classrooms as well as about technology professional development for teachers was reviewed

and a series of introductory workshops for the dynamic mathematics software GeoGebra was evaluated in order to find an answer to the following research question:

Is it possible to identify common impediments that occur during the introduction process of dynamic mathematics software as well as to detect those especially challenging tools and features of the software GeoGebra in order to (a) provide a basis for the implementation of more effective ways of introducing dynamic mathematics software to secondary school mathematics teachers, and (b) to design corresponding instructional materials for technology professional development?

Hence, there are four main objectives for this dissertation, namely . . .

- to identify common impediments teachers face during the introduction process of dynamic mathematics software, and means for preventing such impediments in future GeoGebra introductory workshops,
- to assess GeoGebra's usability, and to identify those challenging features and tools that could potentially cause additional difficulties during the introduction process,
- to establish complexity criteria for dynamic geometry tools as a basis for determining the general difficulty level of such tools in order to facilitate their introduction to novices in future workshops, and
- to design appropriate instructional materials for the successful introduction of GeoGebra that respond to the findings of this study while providing a basis for high-quality professional development in the use of software and technology for secondary school mathematics teachers.

1.3 Structure of the Thesis

Chapter 1

Chapter 1 contains an abstract of the dissertation and explains the objectives of the thesis. Additionally, an overview of the structure is given and the content is summarized.

Chapter 2

Chapter 2 deals with technology professional development for mathematics teachers. In a review of technology professional development, the potential of technology for mathematics education and its integration process into mathematics classrooms are discussed. The design of 'traditional' approaches and suggestions for more successful professional development events are described. Research outcomes as well as deficiencies in research focused on technology in mathematics education and technology professional development are summarized. The second section of this chapter deals with the more or less successful

integration of technology into mathematics teaching and discusses the role teachers play in this complex process. The impact of technology on teaching methods and mathematical content taught is discussed, and a comparison between the envisioned and actual use of technology for teaching and learning mathematics is drawn. In the final section of this chapter, technology used in mathematics education is reviewed. Virtual manipulatives as well as general software tools are described, and different types of software tools used for teaching and learning mathematics are introduced.

Chapter 3

In chapter 3 the dynamic mathematics software GeoGebra is introduced. In the first section background information about the software is presented containing details about its open-source nature and popularity in an international user community as well as a short overview of this software's development. The second section of this chapter discusses the overall design of GeoGebra. After explaining differences between GeoGebra and other software packages used for mathematics teaching and learning in secondary schools, its user interface as well as the implementation of e-learning principles in the design of the software are discussed. In the third section the creation of instructional materials with GeoGebra is summarized. An overview of the basic skills teachers need in order to be able to create their own teaching materials with the software is given, and then the creation of static as well as web-based interactive instructional materials is explained. The final section of this chapter is devoted to teaching mathematics with GeoGebra. After describing the use of GeoGebra as a presentation tool, best practice examples for everyday teaching with the software are given. Finally, GeoGebra's potential to foster discovery learning as well as mathematical explorations and its use for lesson enrichment are described, and examples for student centered teaching of mathematics are presented.

Chapter 4

In chapter 4 the process of introducing GeoGebra and other software packages used for mathematics education is summarized and the research design of this thesis is introduced. In the first section of this chapter the context and environment of the study are described as well as reasons for selecting the dynamic mathematics software GeoGebra for the implementation of this study are given. In the second section introductory materials for dynamic geometry software packages are reviewed and their common characteristics are summarized. The third section of this chapter is devoted to the design of the GeoGebra introductory workshops evaluated for this study. After a summary of the workshops' objectives, an overview about their structure and content is given and their actual implementation as well as teaching methods used by the presenter are described. Detailed information about the workshop content can be found in the appendix (see chapter A). The last section of chapter 4 deals with the evaluation process of these GeoGebra introductory workshops. The research questions are stated and evaluation instruments as well as statistical tests

used in order to analyze the data later on are described. The surveys and questionnaires used for this study can be found in the appendix (see chapter B).

Chapter 5

In chapter 5 a description of the analyzed GeoGebra workshops as well as the workshop participants is given. After summarizing the demographic data of the participants, their general habits concerning computer use both at home and in their schools are analyzed. The second section of this chapter deals with the workshop evaluation and describes how participants rated the workshop activities used to introduce GeoGebra. Additionally, answers to open-ended questions as well as feedback from the participants are summarized. In the last section of this chapter, a summary is supplied of the rating system that was applied to home exercises which participants were supposed to do after each of the four workshop days. After comparing the difficulty ratings of workshop activities and corresponding home exercises, the actual ratings of home exercises are summarized and participant feedback is analyzed. More detailed information about the analysis of open-ended questions and workshop feedback can be found in the appendix (see chapter C).

Chapters 6

Chapter 6 of this dissertation is devoted to the establishment of complexity criteria for dynamic geometry tools. In the first section difficulty level categories for GeoGebra tools are defined which are based on the difficulty ratings given by participants after each workshop. In the second section those dynamic geometry tools introduced in the analyzed GeoGebra workshops are assigned to their corresponding difficulty level categories and their characteristics are described in detail. In the third section of this chapter the complexity of GeoGebra tools is analyzed. After summarizing common characteristics of the tools that were assigned to the same difficulty level category, complexity criteria for dynamic geometry tools are defined and possible reasons for occurring discrepancies between the rated difficulty group and assigned complexity group for certain tools are given. In the fourth section of this chapter all GeoGebra tools are analyzed using the already established complexity criteria for dynamic geometry tools and are assigned to their corresponding complexity group. In the final section of this chapter, the applicability of these complexity criteria for construction tools of other dynamic geometry software is tested by classifying the tools of *Cabri II Plus Geometry* and *Geometer's Sketchpad* according to their functionality. Detailed information about the characteristics and classification of those dynamic geometry tools can be found in the appendix (see chapter D).

Chapter 7

Chapter 7 deals with the evaluation and analysis of other workshop components. In the first section, algebraic input and the use of commands in GeoGebra are analyzed in order to determine their potential impact on the difficulty ratings of workshop activities and

the use of GeoGebra in general. The second section of this chapter summarizes the difficulty ratings of different GeoGebra features that were introduced during the workshops. Thereby, features are grouped by how to access them in GeoGebra, and possible reasons for their difficulty ratings are given. The third section is devoted to the potential impact of external variables on the difficulty ratings of GeoGebra's tools and features as well as the introductory workshops. Participant's gender, age, and teaching experience, as well as mathematics content knowledge, computer skills, and hardware used during the workshops are taken into account. The final section of this chapter summarizes frequently occurring difficulties encountered by the participants while they were observed as documented by helpers who were present in order to assist the participants and give additional support during the workshops.

Chapter 8

Chapter 8 summarizes the research outcomes of this study and suggests further research opportunities based on the study and instructional materials described in this thesis. The first section contains answers to the research questions and summarizes those results relevant for the design of new introductory materials for GeoGebra. In the second section the design of the new GeoGebra documentation is explained which includes the GeoGebra Introductory Workshop Guide for presenters as well as additional workshop materials for participants. Furthermore, design guidelines for dynamic worksheets are introduced which are intended to support teachers when creating their own interactive instructional materials with GeoGebra. The third section is devoted to future professional development with GeoGebra. After describing the structure and objectives of the International GeoGebra Institute, the design of additional GeoGebra documentation is proposed in order to provide ongoing support for mathematics teachers who want to integrate GeoGebra into their teaching practices. The chapter closes with a conclusion for the research study and its potential impact on professional development with GeoGebra.

Chapter 2

Technology in Mathematics Education

This chapter gives an overview of research on technology professional development for mathematics education. After discussing the potential of new technology for education and its integration into teaching practices, the design of professional development events is described and research outcomes and deficiencies are summarized. Then, the integration process of computer technology and its impact on teaching mathematics are discussed and a comparison between the envisioned and actual use of technology for teaching and learning is made. Finally, different types of mathematical software tools and learning environments are presented and introductory materials used for professional development of mathematics teachers are reviewed.

2.1 Review of Technology Professional Development

2.1.1 Potential of Technology for Mathematics Education

Over the last few decades, new technology has become a very important factor in everyday life. Nowadays, computers are vital for business and economy and ‘computer literacy’ is considered a very important skill in our society [Wikipedia, 2008c]. Especially for young people who have grown up having access to computer technology at home, computers have become common tools for communication, text processing, and last but not least, playing games. A multitude of different forms of media are involved, including for example text, audio, graphics, animation, video, and virtual reality [Wikipedia, 2008f]. Additionally, the development and rapid growth of the Internet in combination with its increasing accessibility for the public has opened up a whole new digital world [Wikipedia, 2008e, Wikipedia, 2008d].

Knowing about the increasing importance of new technologies for everyday life, several educational organizations started to develop technology-related standards [Lawless and Pellegrino, 2007, p. 576], trying to foster the integration of new technol-

ogy into teaching and learning. For example, the National Council of Teachers of Mathematics (NCTM), which is the world's largest association of mathematics teachers [Wikipedia, 2008g], declared technology as one of their six principles for school mathematics.

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. [NCTM, 2000, p. 11]

Students can benefit in different ways from technology integration into everyday teaching and learning. New learning opportunities are provided in technological environments, potentially engaging students of different mathematical skills and levels of understanding with mathematical tasks and activities [Hollebrands, 2007, p. 166]. Additionally, the visualization of mathematical concepts and exploring mathematics in multimedia environments can foster their understanding in a new way. Van Voorst [Van Voorst, 1999, pp. 2] reports that technology was "useful in helping students view mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions." Furthermore, he claims that technology helps students to "visualize certain math concepts better" and that it adds "a new dimension to the teaching of mathematics".

Technology environments allow teachers to adapt their instruction and teaching methods more effectively to their students' needs [NCTM, 2000, p. 24]. By integrating educational tools into their everyday teaching practice, they can provide creative opportunities for supporting students' learning and fostering the acquisition of mathematical knowledge and skills. On the one hand, gifted students can be supported more effectively than ever by nurturing their individual interests and mathematical skills. On the other hand, weaker students can be provided with activities that meet their special needs and help them to overcome their individual difficulties. Thereby, students "may focus more intently on computer tasks" and "may benefit from the constraints imposed by a computer environment". Additionally, students can develop and demonstrate deeper understanding of mathematical concepts and are able to deal with more advanced mathematical contents than in 'traditional' teaching environments [NCTM, 2000, p. 24].

2.1.2 Professional Development and Technology Integration

Mathematical skills and knowledge are steadily gaining importance for everyday life in a lot of countries all around the world where "mathematics is viewed as a necessary competency for critical citizenship" [Adler et al., 2005, p. 360]. This heightens the importance of making mathematics education accessible to all students and increasing their mathematics proficiency so as to prepare them for life outside school. In order to provide a higher quality education for students, capable teachers who are willing to implement creative learning environments with technology for the purpose of maximizing their students' learning success are desperately needed [Adler et al., 2005, p. 360].

Being aware of the central role teachers play in their students' learning, professional development for in-service teachers needs to be adapted in order to keep up with the high demands of effectively integrating technology into mathematics teaching. A focus needs to be on fostering students' understanding of mathematical concepts and creating more effective learning environments with technology [The International Commission on Mathematical Instruction, 2004, p. 360 – 361].

During the last 25 years, computer technology for mathematics classrooms experienced an explosive growth both in terms of development as well as availability. This was accompanied by an enormous enthusiasm concerning the potential of new technology for teaching and learning mathematics [Fey et al., 1984]. Consequently, substantial money was invested into equipping schools with hardware, software, and Internet access in order to create an environment that allows technology integration into classrooms [Lawless and Pellegrino, 2007, Cuban et al., 2001]. In 1992, Kaput predicted that, in regard to the continuing growth of electronic technology, “major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures” [Kaput, 1992, p. 515].

During the last two decades researchers became increasingly aware of the important role teachers play for student achievement [Lagrange et al., 2003, p. 257]. They made the implicit assumption that better teacher performance in terms of mathematical content knowledge, pedagogy, and technology integration in combination with knowledge about research outcomes would sufficiently prepare teachers for an easy and effective integration of new technology into their classrooms. Unfortunately, this assumption proved wrong. Many teachers are still struggling with the task of effectively using technology for everyday teaching, and evidence for the predicted improvement of student achievement through effective use of technology for teaching and learning mathematics is still rare. Therefore, researchers became aware of the additional complexity of this process causing them to make more cautious predictions about a successful integration of technology and its impact on mathematics education [Monaghan, 2001].

We acknowledge that we have not explored fundamental questions about teachers' and educators' needs and concerns, but in essence these do not change just because new teaching aids become available. That is, technological advances bring about opportunities for change in pedagogical practice, but do not by themselves change essential aspects of teaching and learning. [Mously et al., 2003, p. 427]

In order to support teachers with the challenge of successfully integrating technology into mathematics teaching and learning, many professional development opportunities were offered that were either adapted to the new tasks, or newly created in order to foster change in teaching practice in the short run and cause improvement of student achievement in the long run [Lawless and Pellegrino, 2007, p. 575]. By actively using technology during professional development events, teachers are supposed to gather different experiences in

terms of learning mathematics than they encountered as students themselves. In this way, they would learn about new approaches of teaching with technology and become better prepared for its effective integration into their own teaching practice [Mously et al., 2003, p. 418]. Additionally, teachers need to learn how to selectively use software in their classrooms, how to increase the interactivity and flexibility of mathematics learning, and how to improve student achievement by providing new and more effective learning opportunities [Mously et al., 2003, p. 395 – 396].

In short, solely providing new technology to teachers does not guarantee its successful integration into mathematics teaching and learning. Appropriate technology professional development needs to be provided in order to support teachers with this task by teaching them not only the use of new software tools but also by introducing them to methods of how to effectively integrate technology into their teaching practices. Furthermore, teachers need to be prepared for the increasing complexity of their teaching environment which definitely creates more challenges for both teachers and students than ‘traditional’ classroom settings.

2.1.3 Design of Professional Development

Traditional Design of Technology Professional Development

Although the importance of teacher professional development has been recognized and the number of training opportunities is increasing, reports indicate that the quality of teacher professional development events is inadequate in general [Ansell and Park, 2003, Edwards, 1997] and that they are not appropriate for preparing teachers sufficiently for a successful technology integration into their classrooms.

Most technology professional development opportunities for in-service teachers are offered in the form of short workshop events [Parsad et al., 2001] that focus on imparting general knowledge about a new educational software with basic skills needed in order to operate it. Due to limited time, many workshops don’t cover how this software could be successfully integrated into teaching mathematics, and therefore, transfer of new skills to their classrooms is made more difficult than necessary for teachers [Knapp, 1996, McCannon and Crews, 2000]. Moreover, teachers are not taught to integrate technology selectively into their teaching practice and that its use should be “limited to those areas where it provides benefits, and reduced in areas where it does not” [Wenglinsky, 1998, p. 33 – 34].

Research indicates that such fragmented approaches to teacher professional development with technology don’t meet the pedagogical needs of teachers, and that the content often is disconnected from everyday classroom practice and teaching methods [Gross et al., 2001, Moursund, 1989]. Therefore, teachers either don’t use the new software at all for their teaching, or they try to apply it to any teaching situation without reflecting on whether or not the software is appropriate for the mathematical content and if their students would benefit more from ‘traditional’ teaching approaches.

Design of More Effective Technology Professional Development

In order to improve the effectiveness of professional development events, the activities and workshops should be spread out over time and include opportunities for follow-up learning as well as feedback and reflection [Lawless and Pellegrino, 2007, p. 594]. Wenglinsky [Wenglinsky, 1998] states that it is important for teachers to have continuous training in terms of technology use in their classrooms. In addition to these workshops, teachers need to be supported over a longer period of time in order to help them integrate the new technology into their classrooms [Cole et al., 2002].

According to the National Foundation for the Improvement of Education [National Foundation for the Improvement of Education, 1996], the quality of professional development events is determined by a set of variables that include:

- Number of contact hours
- Frequency and type of follow-up support
- Level of access to new technologies for teaching and learning
- Active engagement of teachers
- Relevance of the activities to teachers' individual needs
- Use of peer collaboration and community building
- Clear articulation of a common vision for student achievement

Therefore, high-quality professional development needs to provide a higher amount of contact hours, the opportunity for follow-up workshops, and advanced training concerning the use and integration of the new software tool. Additionally, such workshops need to offer access to new technologies that potentially foster teaching and learning of mathematics and provide teachers with meaningful and relevant hands-on activities for their individual teaching situation. Furthermore, professional development needs to promote peer collaboration among teachers while supporting the establishment of a community of mathematics teachers who share the willingness to integrate technology into their classrooms in order to foster their students' learning and understanding of mathematics [Adelman et al., 2002, National Foundation for the Improvement of Education, 1996, Porter et al., 2000, Sparks, 2002].

The following models and approaches promise to be useful for the design of more successful professional development opportunities that foster effective technology integration into teaching and learning of mathematics.

Teachers training teachers has proven to be a promising model for technology professional development. Research indicates that instructors who are in-service teachers themselves understand classroom practices and the demands of teaching mathematics better than other instructors who are not directly connected to schools.

Their guidance and instruction is more relevant and credible to other teachers [Howard et al., 2000] and is accepted more easily causing a greater willingness to try out the new software in their own classrooms.

Best practice examples presented during professional development events make it easier for teachers to transfer new knowledge and skills into their classrooms. Such a ‘design-based’ approach “affords teachers the opportunity to learn how to use specific technologies situated in the context of their curricular needs” [Lawless and Pellegrino, 2007, p. 594]. This workshop design fosters teachers’ taking ownership of the provided resources, and it strengthens their confidence in integrating the new technology into their teaching practices. Additionally, teachers who participated in these workshops are more likely to believe that the software introduced could facilitate their teaching and have a positive impact on their students’ learning of mathematics [Kubitskey et al., 2003].

Accompanying supporting materials are very helpful for teachers who were introduced to a new software during a workshop and want to continue using it. Booklets containing instructions about the basic use of the software, as well as handouts with tips and tricks, can make it easier for teachers to handle the new technology once they are on their own. By providing such materials, teachers can concentrate on the workshop content without having to worry about memorizing every single step or taking notes on paper [Pitcher, 1998, p. 716].

Collaboration with colleagues also facilitates technology integration into teaching practices. Traditionally, most teachers are used to working on their own without having regular meetings with colleagues. By working in teams, teachers can share their knowledge and experience about teaching mathematics in general and about integrating a specific software into their teaching in particular [Lawless and Pellegrino, 2007]. Apart from being able to share instructional materials, teachers can discuss best practice examples, help each other with problems that occurred during their lessons with technology, and share experiences, tips and tricks concerning the general use of a software tool for teaching and learning.

Technical support is another important factor for successful professional development with technology. Although not directly related to the technology workshop itself, teachers need technical support in their schools in order to give software integration into their everyday teaching practices a fair try [Cuban et al., 2001, Holland, 2001]. On the one hand, a school has to provide an appropriate technological infrastructure consisting of hardware components, like computers and projectors, software packages including a pool of general software tools and available online learning resources, as well as a reliable Internet connection. This infrastructure needs to be accessible by teachers and students on a frequent basis. On the other hand, teachers need to have contact persons who are able to deal with technical problems that potentially occur

when using computers and who are available during a school day in order to provide immediate support.

2.1.4 Research Outcomes and Deficiencies

Research on Technology Integration

Technology integration into mathematics teaching and learning provides a very active field of educational research and technology innovations. The great amount of available literature offers a wide range of theories, methodologies, and interpretations, which are often related to the potentialities of new technology for mathematics education. Although the successful integration of technology into mathematics classrooms is a very complex process, researchers tend to tackle very specific aspects instead of trying to understand the process as a whole [Lagrange et al., 2003, p. 237 – 238].

Additionally, most of the research on technology integration is conducted in form of descriptive studies, which contain reports about best practice examples and how they were implemented into a mathematical learning environment. Researchers often describe what went well in a specific situation and for a single teacher in his/her individual situation [Mously et al., 2003, p. 425]. Apart from analyzing classroom learning with technology, many research studies are also conducted in order to find out more about the ‘instrumental genesis’ [Trouche, 2003] (see section 2.3.2) for different types of educational mathematics software: computer algebra systems [Artigue, 2002, Ruthven, 2002], spreadsheets [Haspekian, 2005], and dynamic geometry software [Mariotti, 2002].

In general, research about technology integration into mathematics education is dominated by studies about the innovative use of new technology tools as well as their applications in mathematics education [Lagrange et al., 2003, p. 246]. By contrast, hardly any publications deal with potential difficulties that could occur during the introduction and integration process of technology into everyday teaching and learning of mathematics, or with more established uses of technology in teaching practices, which potentially could help to “gain insights that are better supported by experimentation and reflection” [Lagrange et al., 2003, p. 256].

Research about Technology Professional Development

Although the number of professional development opportunities for in-service teachers is increasing enormously, knowledge about which practices are effective and which don’t work out as anticipated is still lacking [Lawless and Pellegrino, 2007, p. 575]. It seems that research about technology professional development isn’t able to keep up with the quick pace of new technology development [Hamies and Malone, 2001, p. 585] and researchers are far from being able to understand the characteristics of effective professional development.

An increasing pace of change is evident in development of new tools to facilitate the doing of mathematics, the teaching and learning of mathematics, and teacher education itself. As soon as one innovation is engaged with, another

becomes available – but we have little sense of whether all of this activity results in better mathematics classrooms. [Mously et al., 2003, p. 395]

Additionally, knowledge is lacking about what teachers actually learn in professional development events, if and how they are transferring their new knowledge and skills to their classrooms, and if such teacher training events affect students achievement positively [Fishman et al., 2001].

Yet we have much to learn about how to track teachers' knowledge into practice, where knowledge is used to help students learn. And we have more to understand about how teacher education can be an effective intervention in the complex process of learning to teach mathematics, which is all too often most influenced by teachers' prior experiences as learners, or by the contexts of their professional work. [The International Commission on Mathematical Instruction, 2004, p. 361]

Nevertheless, research about technology professional development helped to identify some of the factors that potentially foster a more effective training for in-service teachers which supports successful integration of technology into teaching and learning environments. The following selection describes those factors that should be considered for professional development of mathematics teachers who are struggling to create more effective learning opportunities for their students by integrating technology into their classrooms.

Mathematics content knowledge: Professional development needs to target teachers' mathematical content knowledge [Shulman, 1987, Hill et al., 2005] in order to prepare them for the challenge of dealing with more complex student questions and mathematical enquiries that technology integration is very likely to cause. Since technology allows for mathematical experiments as well as student-centered and active learning, more advanced mathematical content can be covered in mathematics classes. Thereby, 'what happens if...' questions [Fuchs, 2006], mathematical reasoning, and unexpected discoveries are going to be part of everyday teaching, challenging the teachers' mathematical content knowledge and abilities to explain mathematical concepts that couldn't be covered in a 'traditional' classroom setting.

Basic computer literacy: Professional development needs to focus on increasing teachers' general computer literacy and elevating their attitudes as well as confidence concerning computer use for teaching mathematics [Lawless and Pellegrino, 2007, p. 596]. If teachers don't feel comfortable about using technology in front of their students and if they are not able to deal with potentially occurring difficulties and technical problems, then they are less likely to be able to effectively integrate technology into their teaching practice and they will deprive their students of this new opportunity for learning and understanding mathematics. Therefore, teachers need to learn to see computers and educational software "as learning resources and not as ends in themselves" [Mously et al., 2003, p. 419].

Basic software use: Technology professional development needs to help teachers learn the basic use of mathematical software and teach them about possible applications of this particular piece of technology. Research indicates that it is important to know in which way a software package can be introduced to novices most effectively [Mously et al., 2003, p. 401] in order to minimize difficulties and impediments during the introduction process and to facilitate the first contact with the new software tool. Additionally, knowledge about the design of software and its intended applications can help teachers use the technology selectively [Mously et al., 2003, p. 401] and strengthen their own judgement about the appropriateness of software in a certain teaching context. Eventually, teachers need to be prepared for a responsible use of technology in order to foster their students' learning of mathematics by deciding about which type of technology potentially could enhance their students mathematical experiences and understanding of mathematical concepts.

Technology integration: Simply training teachers to use appropriate tools in workshops doesn't guarantee that they are going to use the same technology effectively in their classrooms [Mously et al., 2003, p. 421]. Therefore, professional development events also need to target the transfer of teachers' new knowledge and skills into their classroom practice. On the one hand, teachers need to learn how to integrate new teaching methods into their 'traditional' classroom settings and to use technology effectively but not exclusively. On the other hand, teachers need to be taught how to design new learning activities that tap the full potential of new technology in order to maximize their students' benefit from these new teaching and learning tools [Mously et al., 2003, p. 419].

2.2 Integrating Technology into Mathematics Teaching

2.2.1 Introducing New Technology: Calculators and Computers

With the production of pocket calculators around 1970, the first 'technological revolution' of mathematics teaching and learning was set off. Though initially very expensive, costs of pocket calculators dropped during the next six years, making them affordable for everyone [Wikipedia, 2008a]. Thus, their introduction in schools wasn't delayed much longer and pocket calculators could be legally used by students in the late 1970s. Expectations concerning changes in pedagogy and mathematical content were very high [Weigand and Weth, 2002, p. 4] and the usage of pocket calculators in schools was expected to ...

- ... increase the importance of experimental and discovery learning
- ... strengthen modelling and mathematical concepts

- ... enhance application tasks
- ... reduce the importance of manual computational skills
- ... increase the importance of algorithms

Furthermore, the introduction of pocket calculators raised a lot of pedagogical questions which are very similar to the ones discussed concerning the introduction of computers and mathematical software nowadays [Weigand and Weth, 2002, p. 4].

- How can basic objectives of mathematics education be reached more effectively?
- What is the meaning of ‘traditional’ mathematical skills?
- What are we supposed to do with the additional time gained?
- How is using this new technology going to affect weaker students?

Since many schools and teachers weren’t really prepared for the introduction of this new tool for teaching and learning mathematics, the full potential of pocket calculators couldn’t be tapped at all in the beginning. Nevertheless, new and innovative ideas were implemented and the effective usage of pocket calculators increased in schools over the next decades [Weigand and Weth, 2002, pp. 4]. The use of pocket calculators for teaching and learning mathematics was controversial and caused considerable discussion about the potential loss of computational skills among students [Weigand and Weth, 2002, p. 6]. Although teaching of several mathematical topics was influenced by this new tool, the introduction of pocket calculators didn’t really change objectives, methods, or assessment in mathematics education [Weigand and Weth, 2002, Fey and Hirsch, 1992].

In general, the process of introducing new technology in the form of pocket calculators in schools showed that just providing a new tool along with several best practice examples couldn’t change mathematics education fundamentally. Instead, teacher education and professional development needed to be changed as well in order to prepare teachers for this new methodological tool and teach them how to effectively integrate it into their everyday teaching. The conclusion drawn from experience was that the full potential of new technology can only be fully realized if teachers are convinced of its benefits for teaching and learning mathematics [Weigand and Weth, 2002, p. 6].

With a delay of about 10 years, personal computers followed the pocket calculators into schools. With regard to their expected impact on future everyday life, ‘computer literacy’ became an important keyword, and schools were supposed to prepare students for this new challenge. The focus lay on familiarizing students with computers in general while teaching the basic use often thrust mathematical contents aside.

In the late 1980s pedagogical aspects gained in importance thereby causing a call for more user-friendly software in order to allow for focusing on content instead of the technology itself [Weigand and Weth, 2002, p. 6 – 7]. Meaningful integration of new technology into teaching became the general objective, which was supported by the development of

the first dynamic geometry software *Cabri Geometry* and the computer algebra system *Derive* [Weigand and Weth, 2002, p. 7 – 8]. Additionally, drill-and-practice programs and computer-assisted instruction [Kaput, 1992, Kaput and Thompson, 1994], which were the first applications of computers for mathematics learning [Hurme and Järvelä, 2005, p. 50], were increasingly replaced by multimedia learning environments. Thus, the use of technology as a cognitive tool in order to allow students to construct individual knowledge was fostered [De Corte et al., 1996].

Again, discussions about effective use of these new tools took place and questions about selective integration of new technology into teaching and its role for assessment were raised. However, expectations concerning potential changes of mathematics education in terms of objectives, contents, and instructional methods were more realistic this time: new technologies were supposed to be successively integrated into teaching and learning, supporting an ‘evolution’ instead of causing a ‘revolution’ [Weigand and Weth, 2002, p. 10].

2.2.2 Teachers’ Role in the Integration Process

Although today, access to new technology is provided in most schools [Cuban et al., 2001, p. 815], the process of technology integration into everyday teaching is still very slow, and the full potential of computers and software for mathematics teaching and learning is far from being tapped. Among the various reasons research has found for this phenomenon, NCTM expressed that which is probably the most crucial one in *Principles and Standards for School Mathematics* [NCTM, 2000, p. 25].

The effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students’ learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well — graphing, visualizing, and computing.

But what are possible reasons for teachers’ failure to use such powerful tools which have the potential to facilitate their everyday teaching and to provide numerous benefits for their students? Wenglinsky summarizes the challenges for teachers related to the integration of a new tool into teaching as follows:

[T]eachers have historically been resistant to technological innovations when those innovations have made it more difficult for them to get through the typical school day. [Wenglinsky, 1998, p. 8]

Using computers and learning how to work with special software definitely is a challenge for teachers, especially, if they have no experience with new technology. Once they have mastered the basic skills necessary to operate software, there is still a long way to go before they actually are able to effectively integrate it into their everyday teaching practice.

Impediments caused by the time management and organization of schools, as well as external standardized tests, problems with the hardware, software, and Internet connection, limited access to school computers, and lack of communication and collaboration between teachers [Cuban et al., 2001, p. 828 – 829] combined with lack of support from school administration make it difficult for teachers to use new technology in their classrooms.

However, the first step to support teachers in this situation should be to teach them the basic use of appropriate software and increase their comfort level concerning its potential applications in their classrooms. By providing prepared instructional materials, teachers can get used to the idea of integrating software into their classroom practices and teaching methods without having to spend additional time on creating materials and generating ideas on how to effectively use technology for their teaching. Thus, teachers can focus on potentially modifying their teaching methods and broaden their instructional repertoires in order to provide more effective learning opportunities for their students in ways that wouldn't be possible without technology. By helping teachers to treat technology as an already developed educational tool and allowing them to focus on the teaching of mathematics itself, the integration of technology into everyday teaching could be facilitated in a way that would allow teachers and students to benefit from their new, technology enhanced teaching and learning environment.

2.2.3 Impact on Teaching Methods and Content

New technologies offer new ways of dealing with traditional content in many mathematical areas. [Hohenwarter, 2006a, p. 5]

In the *Principles and Standards for School Mathematics*, NCTM states that “[t]echnology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum” [NCTM, 2000, p. 25]. Therefore, instructional methods and also the mathematical content taught need to be adapted in order to foster teachers’ effective use of new technological tools for teaching and learning. Also, Lawless and Pellegrino [Lawless and Pellegrino, 2007, p. 581] support this point of view stating that “[t]echnology can make it quicker or easier to teach the same things in routine ways”. Additionally, they suggest that technology allows teachers to “adopt new and arguably better approaches to instruction and/or change the content or context of learning.”

Whenever technology is used for teaching mathematics, it is the responsibility of the teacher to decide when technology can effectively improve learning opportunities and which kind of technology is appropriate to reach objectives of the lesson [Lawless and Pellegrino, 2007, p. 581]. Accordingly, “technology should be used widely and responsibly, with the goal of enriching students’ learning of mathematics” [NCTM, 2000, p. 24].

Since technology allows for more student-centered approaches including active learning, mathematical experiments, or discovery learning [Bruner, 1961], usually the role of a teacher needs to transform from being instructor to being a coach or mentor for students.

Although students often can work independently of the teacher when using computers and appropriate software, new technology never will be able to replace teachers, since they play vital roles in technology-rich classroom settings [NCTM, 2000, p. 25]. Thereby, technology potentially opens up new observation possibilities for teachers, allowing them to focus on students' investigations and thinking strategies while solving mathematical problems [NCTM, 2000, p. 25].

Additionally, technology can also impact the mathematical content accessible to students as well as modify the tasks used to deliver that content. For example, Laborde [Laborde, 2001, p. 293] proposes to classify mathematical tasks according to the role that a dynamic geometry software like *Cabri Geometry* plays in the instructional process. She distinguishes between four types of roles: At first, the software could facilitate "material aspects of the task while not changing it conceptually". Secondly, the software could be used as a 'visual amplifier' [Pea, 1985] in order to facilitate observations, such as for example identifying properties of geometric figures. Thirdly, by providing special tools the software allows students to solve mathematical tasks in new ways. Finally, a dynamic geometry software allows for creation of a new type of mathematical problems which couldn't be treated in classrooms without technology, like for example dynamic investigations of mathematical concepts.

2.2.4 Vision versus Reality

While "[a]dvocacy of the educational use of new technologies often seems to suggest that their value is evident, their adoption urgent, their implementation unproblematic, and their impact transformative" [Ruthven, 2006, p. 161], integrating new technology into everyday teaching and learning of mathematics has proven to be a slow process that involves multiple challenges for teachers and students [Hohenwarter and Lavicza, 2007, p. 49]. Teachers not only need adequate training and experience using software, but they also need time to accept and adapt to the changes necessary to effectively integrate technology into their classrooms including changes of teaching methods, learning situations, and also mathematical concepts and contents taught [Laborde, 2001, p. 311].

Although the availability of electronic resources is increasing steadily and most students and teachers have access to computers in schools as well as at home [Cuban et al., 2001], technology is integrated poorly into the instructional practices of teachers and classroom activities [Lawless and Pellegrino, 2007, p. 580 – 581]. Computers are most frequently used during classes for word processing and practicing basic skills, while engaging students into analytical thinking and problem solving through simulations and other media is rather rare in everyday teaching [Becker, 2000, Hart et al., 2002].

According to Rogers [Rogers, 1995] the phenomenon of slow integration of technology into teaching and learning can be explained with the 'slow revolution' idea. It states that small changes accumulate over time and create a very slow transformation within an existing system. This is based on the notion of time lag between the invention of new technology, its adoption by individuals, and the following slow distribution process among

the general public. Individuals might need years or even decades in order to learn the basic use of the new technology.

However, in the case of technology integration into teaching and learning, the ‘slow revolution’ hasn’t occurred yet [Cuban, 1986]. Although access to technology is given in most schools, many teachers lack the knowledge of how to use it and successfully integrate it into their teaching practice [Swain and Pearson, 2002, Lawless and Pellegrino, 2007]. Many teachers don’t feel comfortable with using computers for teaching or they do not understand how using technology could possibly assist in providing engaging and meaningful learning environments for their students [Niederhauser and Stoddart, 1994]. Furthermore, practical constraints such as not having access to enough computers in order to supply a whole class of students [Mously et al., 2003, p. 420], or having to deal with unreliable hardware and not being supported sufficiently by technical support staff [Cuban et al., 2001, p. 829] contribute to teachers reluctance concerning the use of new technology.

2.3 Technology Used in Mathematics Education

In general, technology is integrated into mathematics teaching and learning in two forms. On the one hand, there are virtual manipulatives which consist of specific interactive learning environments that usually can be accessed online. In the virtual manipulatives setting students can explore mathematical concepts without having special computer skills or knowledge about specific educational software packages. On the other hand, there are mathematical software tools that are appropriate for educational purposes and can be used for a wide variety of mathematical content topics.

2.3.1 Virtual Manipulatives

Definition and Potential

[...A] virtual manipulative is best defined as an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge. [Moyer et al., 2002, p. 373]

Today, the Internet provides a large variety of different virtual manipulatives for mathematics education in the form of ‘applets’, ‘mathlets’, or ‘dynamic worksheets’¹. Their potential for improving the quality of mathematics teaching and learning is promising as virtual manipulatives support students’ active learning and potentially foster their conceptual understanding and problem solving skills. Additionally, students can gather positive experience with mathematical experiments which increases their motivation to deal with mathematical contents [Durmus and Karakirik, 2006].

Virtual manipulatives [...] may provide interactive environments where students could pose and solve their own problems to form connections be-

¹The term ‘dynamic worksheets’ is specifically used for virtual manipulatives created with GeoGebra.

tween mathematical concepts and operations, and get immediate feedback about their actions that might lead them to reflect on their conceptualization. [Durmus and Karakirik, 2006, p. 5]

A special advantage of virtual manipulatives compared to their ‘real’ counterparts is that they are mostly available online, and therefore, accessible for students both in school and at home. Additionally, they are available in literally any number allowing teachers to equip students of all class sizes and reduce the time-consuming activities of distributing manipulatives before and tidying them up after their use. Therefore, teachers are more likely to give their students assignments that involve mathematical explorations and discoveries [Moyer et al., 2002, p. 375].

Using virtual manipulatives, impediments related to the quantity of available manipulatives, as well as classroom organization and time management could be prevented. Furthermore, the notion of students in upper grade levels that manipulatives are ‘just for little kids’ probably doesn’t apply to virtual manipulatives, because “[o]lder students may view the use of virtual manipulatives on the computer as more sophisticated than using manipulatives in their concrete form” [Moyer et al., 2002, p. 375 – 376].

Nowadays, there are extensive pools of virtual manipulatives available on the Internet. The “best virtual manipulative sites are those with a variety of dynamic features that allow users to perform various mathematical investigations” [Moyer et al., 2002, p. 374]. Additionally, such interactive learning environments often link different representations of mathematical objects (e.g. graphic and symbolic), offer links to other online resources, allow users to modify the objects available (for example, they might be allowed to increase their number according to the individual solution strategy), or change the color of objects in order to facilitate the exploration process [Moyer et al., 2002, p. 375].

In order to support teachers who want to integrate virtual manipulatives into their teaching environment, Clements and McMillen established recommendations for teachers that are specific to the use of virtual manipulatives for teaching and learning mathematics [Clements and McMillen, 1996, p. 278]:

- Use computer manipulatives for assessment as mirrors of students’ thinking.
- Guide students to alter and reflect on their actions, always predicting and explaining.
- Create tasks that cause students to see conflicts or gaps in their thinking.
- Have students work cooperatively in pairs.
- If possible, use one computer and a large-screen display to focus and extend follow-up discussions with the class.
- Recognize that much information may have to be introduced before moving to work on computers including the purpose of the software, ways to operate the hardware and software, mathematics content and problem solving strategies, and so on.
- Use extensible programs for long periods across topics when possible.

Quality and Flexibility

Most virtual manipulatives are small self-contained learning environments that focus on specific mathematical topics. Therefore, “it might be argued some of the developed manipulatives lack the desired level of interactivity, usability and motivation” [Durmus and Karakirik, 2006, p. 6]. Although most of them are ready-to-use for demonstration by teachers and / or self-directed learning by students, they often do not exactly match the needs of users and can’t be changed at all or only with significant effort and advanced computer or even programming skills. Additionally, the modification of virtual manipulatives may also constitute a violation of its author’s copyright [Hohenwarter and Preiner, 2007a, p. 2].

However, there are good reasons why it seems desirable to be able to change existing interactive teaching materials. On the one hand, the quality of mathlets is quite diverse as only a small portion of them is reviewed or edited before getting published on the Internet [...]. By being able to change such materials, educators can work together to improve their quality. On the other hand, someone may want to add missing features or remove unneeded parts for their specific purposes. Basing this work on existing mathlets can save a lot of time and effort. [Hohenwarter and Preiner, 2007a, p. 2]

Nevertheless, there are exceptions to this phenomenon, and these exceptions may be represented by ‘dynamic worksheets’ that were created with the dynamic mathematics software *GeoGebra* and published on the GeoGebraWiki². These interactive webpages not only can easily be edited using an html editor, but also the interactive applet can be modified using GeoGebra. Therefore, this kind of virtual manipulatives is much more flexible and can be adapted to students’ needs by their own teacher.

For all other virtual manipulatives available on the Internet, teachers need to determine the quality level and usability prior to using them in their classrooms. Clements and McMillen [Clements and McMillen, 1996, p. 277] developed a set of recommendations and special considerations that could help teachers to pick beneficial virtual manipulatives from the large pool provided on the Internet. They recommend the selection of virtual manipulatives that ...

- have uncomplicated changing, repeating, and undoing actions;
- allow students to save configurations and sequences of actions;
- dynamically link different representations and maintain a tight connection between pictured objects and symbols;
- allow students and teachers to pose and solve their own problems; [...]
- allow students to develop increasing control of a flexible, extensible, mathematical tool[;...]

²Pool of free virtual manipulatives, www.geogebra.org/en/wiki

- encourage easy alterations of scale and arrangement;
- go beyond what can be done with physical manipulatives; and
- demand increasingly complete and precise specifications.

2.3.2 General Software Tools

[W]e need software where children have some freedom to express their own ideas, but constrained in ways so as to focus their attention on the mathematics. [Hoyles, 2001]

Although using virtual manipulatives might be convenient for teachers, their limitation of mathematical experiments to a certain range of activities and topics are obvious. Therefore, many teachers (also) use educational software packages that allow more flexibility and enable both teachers and students to visualize and explore mathematical concepts in their own creative ways [Barzel, 2007].

General tools for mathematics education include for example dynamic geometry software, computer algebra systems, spreadsheets, and dynamic mathematics software. Fuglestad [Fuglestad, 2005] defines such tools

[...] as open and flexible software, not made for specific topics or limited to teach specific tasks. [This kind of] computer software [...] makes it possible for the user to plan and decide what to do. Such tools can be used for a wide variety of problems and can provide learning situations to explore and experiment with mathematical connections, and provide new ways of approaching the tasks. [Fuglestad, 2005, p. 1]

According to the ‘instrumental approach’ [Guin and Trouche, 1999] general software tools are artifacts which need to be transformed into mathematical instruments by students in order to successfully use them for solving given tasks.

The instrumental dimension of IC technologies distinguishes artifact and the instrument a human being is able to build from this artifact. While the artifact refers to the objective tool, the instrument refers to a mental construction of the tool by the user. (original [Chevallard, 1999], translation [Trouche, 2003, p. 785])

Thus, a mathematical software tool per se doesn’t have the power to improve mathematical learning, but can be transformed by a user into a mathematically useful instrument. This process is called ‘instrumental genesis’ [Trouche, 2003, Trouche, 2004] and can be rather long and complex [Artigue, 2002] since the same artifact can be transformed into a variety of tools depending on the user and the task to be solved [Trouche, 2002]. Thereby, characteristics of the artifact, including its potential and constraints, as well as the skills, knowledge, and experiences of the user need to be taken into account [Trouche, 2003, p. 786].

Integration of General Tools into Classrooms

Although “[g]eneral tools allow students and teachers much more freedom to shape and modify how to use them” [Barzel, 2007, p. 80], the introduction of a general tool for mathematics education requires more time and effort from both teachers and students than simply using virtual manipulatives. While students need hardly any computer skills in order to be able to work with prepared virtual manipulatives, both teachers and students need to learn basic skills concerning the operation of a specific mathematics software before being able to effectively integrate it into teaching and learning. Although many teachers don’t want to spend this time to introduce a software package to their students, teaching them the basic use of the tool usually pays off in the long run, and especially if the software package is a versatile tool that allows teachers to cover different mathematical topics at different grade levels.

However, providing students with a mathematical software, and the basic skills necessary to operate the tool, doesn’t guarantee an effective use and benefits for the learning and understanding of mathematics. Instead, teachers need to emphasize selective use of mathematical software tools by answering and discussing questions like the ones below.

- Does it make sense to use software in order to solve a given task or is it more effective to use ‘traditional’ tools like paper, pencil, straightedge, and compass?
- Which software is appropriate for which mathematical content?
- Which features of the software have to be familiar to the students?
- How can the software be effectively used in order to solve a given task?
- Which kind of guidance and instructions do students need in order to be able to successfully solve a given task?

According to Brown et al. [Brown et al., 2009, p. 4]

[i]nstrumental integration is a means to describe how the teacher organizes the conditions for instrumental genesis of the technology proposed to the students and to what extent (s)he fosters mathematics learning through instrumental genesis. It rests on two main characteristics of the teaching situation. The first is the know-how of the pupils regarding the artefact. The second is the didactical aim of the tasks given to the pupils.

In this context the authors distinguish between four modes of technology integration into mathematics teaching which are determined by (a) the technology know-how of the students concerning the use of a special mathematical software as well as by (b) the nature of tasks and educational objectives that should be reached [Brown et al., 2009, p. 4 – 5].

Instrumental initiation: Since students are novices to the software, the teacher’s main objective is to introduce them to the software and teach them the basic skills needed. Thereby, tasks focus on the technology, and the level of instrumental integration of the tool into learning of mathematical content is minimal.

Instrumental exploration: Although students are not familiar with the use of a particular software package, the teacher wants them to increase their mathematical knowledge while exploring the technological tool. Thus, students “explore the technology through mathematical tasks” [Brown et al., 2009, p. 5]. Thereby, the instrumental integration depends on the mathematical task as well as on guidance and instructions provided by the teacher.

Instrumental reinforcement: As students are already familiar with the basic use of a particular software package, they might experience technical difficulties solving a given task. The teacher provides specific additional information about the corresponding feature(s) of the software in order to help them overcome their problems. Nevertheless, the teacher’s main objective is to increase the students’ mathematical knowledge. Thereby, the instrumental integration varies depending on the kind of support the teacher provides.

Instrumental symbiosis: Students who have a certain expertise using a software work on tasks that allow them to both improve their technical skills and mathematical knowledge. Therefore, the instrumental integration is maximal in this setting.

2.3.3 Types of Software Tools Used for Mathematics Education

Computer algebra systems, dynamic geometry software, and spreadsheets are the main types of educational software currently used for mathematics teaching and learning [Drijvers and Trouche, 2007, Fuglestad, 2005, Leuders et al., 2005]. Each of the programs has its own advantages and is especially useful for treating a certain selection of mathematical topics or supports certain instructional approaches. Nevertheless, the boundaries between those types of software become increasingly blurred and features characteristic for one type are often added to another one. Thus, a new type of educational software, so called dynamic mathematics software, was designed with the purpose to join the advantages of different types of mathematics software so as to become a versatile tool for mathematics teaching and learning that can be used for a wider range of mathematical contents, grade levels, and teaching methods.

Computer Algebra Systems

Computer algebra systems (CAS) are designed to facilitate the manipulations of mathematical expressions in symbolic form [Wikipedia, 2008b]. Examples of computer algebra systems are *Derive* [Texas Instruments Inc., 2008], *Maple* [Maplesoft, 2008], and *Mathematica* [Wolfram Research Inc., 2008].

In general, computer algebra systems mainly deal with the symbolic and numeric representation of mathematical objects. They allow for manipulating a variety of algebraic expressions and functions, and can deal for example with basic mathematical operations, simplification, factorization, derivatives, integrals, sequences, and matrices [Fuchs, 2007, pp. 171]. Additionally, they allow for plotting graphs of functions and equations. Thereby,

computer algebra systems are usually operated using keyboard input and enable users to implement their own algorithms using commands and a special syntax [Wikipedia, 2008b]. Additionally, most computer algebra systems allow for graphically displaying explicit and sometimes even implicit equations, whereby those graphical representations usually can't be modified directly by using the mouse [Hohenwarter, 2002, p. 3].

Dynamic Geometry Software

“Dynamic Geometry Software (DGS)” [...] is used as a generic term to describe a certain type of software which is predominantly used for the construction and analysis of tasks and problems in elementary geometry. [Sträßer, 2002]

Pure dynamic geometry software is operated mainly with the mouse by activating different geometric tools and applying them to the drawing pad or already existing objects. Examples of dynamic geometry software are *Cabri Geometry* [Cabrilog SAS, 2007] and *Geometer's Sketchpad* [Key Curriculum Press, 2008]. Although those programs slightly differ in terms of their functionality and use, in general dynamic geometry software provides three main features that usually can't be found in computer algebra systems or spreadsheets: drag mode, customizable tools, and trace or locus of objects [Graumann et al., 1996, p. 197].

Drag mode: Dynamic geometry software allows the creation of geometric constructions and other dynamic figures (e.g. function graphs) by using the computer mouse and a variety of geometric tools and menu items. Relations and dependencies between objects are maintained while an object is dragged with the mouse by updating their positions dynamically. The so called *drag test* is an important concept that enables users not only to check the robustness of a construction by dragging different objects with the mouse, but also to explore a variety of similar constructions and special cases which is not possible in a traditional paper-and-pencil construction.

Apart from dynamic movements, a DGS also allows the user to apply transformations to objects and measure lengths and angles. Additional features include the insertion of text and sometimes images into the drawing pad, which can be used to enhance a dynamic construction.

Customized tools: The available geometric tools are usually organized in toolboxes and can be activated by clicking on the corresponding icon in the toolbar or by selecting appropriate commands from the menu. Additionally, a sequence of construction steps can be grouped and saved as a new tool. Thus, users can define their own geometric construction tools and save them in the toolbar.

Trace or locus: The trace of an object in respect to a parent object can be displayed allowing users to examine movements and dependencies between mathematical objects. In this way, the locus line can either be created manually by moving corresponding objects with the mouse, or created automatically by the software itself.

Dynamic geometry software usually provides the following basic mathematical objects: points, segments, lines, circles, vectors, and conic sections. Additionally, it is possible (a) to do analytic geometry using a coordinate system, and (b) to work with function graphs by creating the locus of a given point whose y -coordinate is calculated using a given expression.

Although keyboard input of numbers and expressions is possible in most dynamic geometry software programs, it is usually limited to a range of special commands and pre-defined expressions. Such input is mainly used to carry out calculations whose results can be integrated into the construction process.

Since the start more than two decades ago, DGS has become one of the most widely used pieces of software in schools and colleges all over the world. [...] If viewed in terms of research on software use in teaching and learning mathematics, DGS may be one of the best (if not the best) researched type of software within Didactics of Mathematics [...]. [Sträßer, 2002, p. 65]

Spreadsheets

Spreadsheets build an ideal bridge between arithmetic and algebra and allow the student free movement between the two worlds. Students look for patterns, construct algebraic expressions, generalize concepts, justify conjectures, and establish the equivalence of two models as intrinsic and meaningful needs rather than as arbitrary requirements posed by the teacher. [Friedlander, 1998, p. 383]

Spreadsheets are computer applications that allow the display of alphanumeric text or numeric values in table cells which are organized in rows and columns. Formulas can be used to calculate new values by referring to other cells. Whenever the content of one cell is modified, all other related cells are updated automatically [Wikipedia, 2008h]. Therefore, electronic spreadsheets are principally used as tools for mathematical and statistical calculations, allowing “students to focus on the mathematical reasoning by freeing them from the burden of calculations and algebraic manipulations” [Ozgun-Koca, 2000].

Spreadsheets are usually operated using keyboard input, formulas, and commands. They allow for plotting data in different types of charts which automatically adapt to modifications of the data. Examples of spreadsheets are *MS Excel* [Microsoft Corporation, 2007] and *Calc* [CollabNet Inc., 2008a].

Dynamic Mathematics Software

In 2000 Schumann and Green stated that

[t]here is a need for further software development to provide a single package combining the desired features [of DGS and CAS]. [Schumann and Green, 2000, p. 337]

Dynamic mathematics software is designed to combine certain features of dynamic geometry software, computer algebra systems, and also spreadsheets into a single package. The resulting new dynamic mathematics software packages differ in their range of combined features, as well as in the degree of dynamic interaction between those features. Examples of dynamic mathematics software are *GeoGebra* [Hohenwarter, 2008] and *GEONE_{xt}T* [Universität Bayreuth, 2007].

In the case of *GeoGebra*, different representations of the same mathematical object are connected dynamically, allowing users to go back and forth between them thereby making relationships among those representations more easily comprehensible for students [Ozgun-Koca, 2000]. Whenever one of the representations is modified, all others adapt automatically in order to maintain the relations between the different objects. New objects can be created either by using dynamic geometry tools or algebraic keyboard input. By its provision for keyboard input, a range of pre-defined commands can be used in *GeoGebra* and mathematical topics other than geometry can be treated as well (e.g. algebra, calculus).

Chapter 3

The Dynamic Mathematics Software GeoGebra

In this chapter the dynamic mathematics software GeoGebra is introduced. After giving some background information about the software and its development, the most important characteristics of GeoGebra that make it distinguishable from pure dynamic geometry software are summarized. The user interface of the software is explained and the implementation of e-learning principles in GeoGebra is described. Since GeoGebra can also be used to create instructional materials, an overview of basic skills needed and the types of instructional materials whose creation is supported by GeoGebra is given. Finally, ways of integrating GeoGebra into everyday teaching are summarized and several best practice examples to illustrate how this could be implemented in a classroom setting are presented.

3.1 Background Information about GeoGebra

3.1.1 What is GeoGebra?

GeoGebra is dynamic mathematics software (DMS) designed for teaching and learning mathematics in secondary school and college level. The software combines the ease of use of a dynamic geometry software (DGS) with certain features of a computer algebra system (CAS) and therefore, allows for bridging the gap between the mathematical disciplines of geometry, algebra, and even calculus [Hohenwarter and Preiner, 2007b]. On the one hand, GeoGebra can be used to visualize mathematical concepts as well as to create instructional materials. On the other hand, GeoGebra has the potential to foster active and student-centered learning by allowing for mathematical experiments, interactive explorations, as well as discovery learning [Bruner, 1961].

GeoGebra is open source software under the GNU General Public License¹ and freely available at www.geogebra.org. Thereby, either an installer file can be downloaded, or GeoGebra can be launched directly from the Internet using GeoGebra *WebStart*. Since the

¹GNU General Public License: www.gnu.org/copyleft/gpl.html

software is based on Java, it is truly platform independent and runs on every operating system. Furthermore, GeoGebra is multilingual not only in its menu, but also in its commands, and was translated by volunteers from all over the world into more than 35 languages (GeoGebra 3.0).

GeoGebra's popularity is growing rapidly all around the world. Currently (March 2008) its webpage receives about 300,000 visitors per month from 188 different countries. The program's developer estimates that more than 100,000 teachers already use GeoGebra all over the world for teaching mathematics and creating static as well as interactive instructional materials to enhance their students' learning.

Probably the most popular question concerning GeoGebra, which is posed during almost every presentation and workshop regardless its location, is "Why is it free?". This characteristic of the software seems to fascinate people, sometimes arousing suspicion among teachers, who fear to be charged later on after they got 'hooked up' with GeoGebra. In the postscript of an article called *Linking Geometry and Algebra with GeoGebra*, the developer himself provides an answer to this question:

GeoGebra is free software because I believe that education should be free. This philosophy makes it easy to convince teachers to give this tool a try, even if they haven't used ICT in their classrooms before. Moreover, some of them translate the software into other languages, share their own materials on the web [...] and answer questions in the user forum [...] – for free, of course. [Edwards and Jones, 2006, p. 30]

3.1.2 Short History of GeoGebra

The development of GeoGebra began in 2001 as Markus Hohenwarter's Master's thesis project at the University of Salzburg, Austria. After studying mathematics education as well as computer engineering, he started to implement his idea of programming a software that joins dynamic geometry and computer algebra, two math disciplines that other software packages tend to treat separately. His main goal was to create an educational software that combines the ease of use of a dynamic geometry software with the power and features of a computer algebra system, which could be used by teachers and students from secondary school up to college level. After publishing a prototype of the software on the Internet in 2002, teachers in Austria and Germany started to use GeoGebra for teaching mathematics, which was, at this point, rather unexpected by the creator, who got a lot of enthusiastic emails and positive feedback from those teachers [Hohenwarter and Lavicza, 2007, p. 49 – 50].

In 2002, Hohenwarter received the European Academic Software Award EASA in Ronneby, Sweden, which finally inspired him to go on with the development of GeoGebra in order to enhance its usability and extend its functionality. Further development of GeoGebra was funded by a DOC scholarship awarded to Hohenwarter by the Austrian Academy of Sciences, which also allowed him to earn his PhD in a project that examined pedagogical applications of GeoGebra in Austrian secondary schools. During the next four years

GeoGebra won several more software and media awards in different European countries, including Austria, Germany, and France [Hohenwarter, 2005, p. 2].

Since 2006, GeoGebra's ongoing development has continued at Florida Atlantic University, USA, where Hohenwarter works in a teacher training project funded by the National Science Foundation's Math and Science Partnership initiative. During the last two years of close collaboration with a number of middle and high school mathematics teachers, GeoGebra was enhanced by including a range of important features. This enhanced functionality enabled the creation of user defined tools and significant simplification in the steps required for user creation of interactive instructional materials, the so called *dynamic worksheets*.

Future plans to further extend and enhance GeoGebra involve the implementation of a dynamically linked spreadsheet, as well as a computer algebra extension, pushing the software further towards the goal of being a versatile and easy to use software package that can be used for a wide range of different grade levels and mathematical contents by students and teachers around the world.

3.2 Design of GeoGebra

3.2.1 Why is GeoGebra different?

Currently, there are two types of educational software that connect the mathematical fields of geometry and algebra and are used for mathematics teaching and learning. On the one hand, there is dynamic geometry software (DGS) that allows users to create and dynamically modify Euclidian constructions. Geometric properties and relations between objects used within a construction are maintained because manipulating an object also modifies dependant objects accordingly. Some dynamic geometry programs even provide basic algebraic features by displaying the equations of lines or conic sections, as well as other mathematical expressions which usually can't be modified directly by the user. On the other hand, there are computer algebra systems (CAS) which symbolically perform algebra, analytic geometry, and calculus. Using equations of geometric objects, a computer algebra system can decide about their relative position to each other, and display their graphical representations. Many computer algebra systems are also able to plot explicit and sometimes even implicit equations. Generally, the geometric representation of objects can't be directly modified by the user.

GeoGebra is an attempt to join these two types of software, whereby geometry, algebra, and calculus are treated as equal partners. The software offers two representations of every object: the numeric algebraic component shows either coordinates, an explicit or implicit equation, or an equation in parametric form, while the geometric component displays the corresponding solution set [Hohenwarter, 2002, p. 3 – 4]. In GeoGebra both representations can be influenced directly by the user. On the one hand, the geometric representation can be modified by dragging it with the mouse, whereby the algebraic representation is changed dynamically. On the other hand, the algebraic representation can be changed using the keyboard causing GeoGebra to automatically adjust the related geometric representation.

This new bidirectional dynamic connection between multiple representation of mathematical objects opens up a wide range of new application possibilities of dynamic mathematics software for teaching and learning mathematics while fostering student understanding of mathematical concepts in a way that was not possible several years ago.

[. . .] There is no other ways of gaining access to mathematical objects but to produce some semiotic representations. [. . .] There is no true understanding in mathematics for students who do not incorporate into their cognitive architecture the various registers of semiotic representations used to do mathematics. [Duval, 1999, p. 4 and 25]

3.2.2 GeoGebra's User Interface

Since GeoGebra joins dynamic geometry with computer algebra, its user interface contains additional components that can't be found in pure dynamic geometry software. Apart from providing two windows containing the algebraic and graphical representation of objects, components that enable the user to input objects in both representations as well as a menubar are part of the user interface (see figure 3.1).

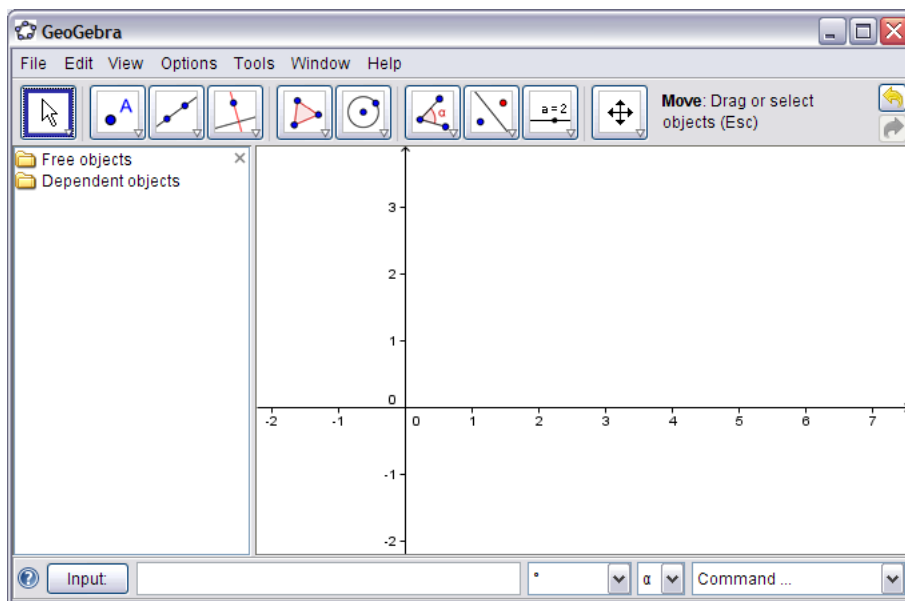


Figure 3.1: GeoGebra's user interface

Graphics window: The graphics window is placed on the right hand side of the GeoGebra window. It contains a drawing pad on which the geometric representations of objects are displayed. The coordinate axes can be hidden and a coordinate grid can be displayed by the user. In the graphics window, existing objects can be modified

directly by dragging them with the mouse, while new objects can be created using the dynamic geometry tools provided in the toolbar.

Toolbar: The toolbar consists of a set of toolboxes in which GeoGebra's dynamic geometry tools are organized. Tools can be activated and applied by using the mouse in a very intuitive way. Both the name of the activated tool as well as the toolbar help, which is placed right next to the toolbar, give useful information on how to operate the corresponding tool and, therefore, how to create new objects. In the right corner of the toolbar the *Undo* and *Redo* buttons can be found, which enable the user to undo mistakes step-by-step.

Algebra window: The algebra window is placed on the left hand side of the GeoGebra window. It contains the numeric and algebraic representations of objects which are organized into two groups:

- *Free objects* can be modified directly by the user and don't depend on any other objects.
- *Dependant objects* are the results of construction processes and depend on 'parent objects'. Although they can't be modified directly, changing their parent objects influences the dependant objects. Additionally, the definition of a dependant object can be changed at any time.

Additionally, both types of objects can be defined as *auxiliary objects*, which means that they can be removed from the algebra window in order to keep the list of objects clearly arranged.

Algebraic expressions can be changed directly in the algebra window, whereby different display formats are available (e.g. Cartesian and polar coordinates for points). If not needed, the algebra window can be hidden using the *View* menu.

Input field: The input field is placed at the bottom of the GeoGebra window. It permits the input of algebraic expressions directly by using the keyboard. By this means a wide range of pre-defined commands are available which can be applied to already existing objects in order to create new ones.

Menubar: The menubar is placed above the toolbar. It provides a wide range of menu items allowing the user to save, print, and export constructions, as well as to change default settings of the program, create custom tools, and customize the toolbar.

Construction protocol and Navigation bar: Using the *View* menu, a dynamic construction protocol can be displayed in an additional window. It allows the user to redo a construction step-by-step by using the buttons of a navigation bar. This feature is very useful in terms of finding out how a construction was done or finding and fixing errors within a construction. The order of construction steps can be changed

as long as this doesn't violate the relations between dependant objects. Furthermore, additional objects can be inserted at any position in order to change, extend, or enhance an already existing construction.

Additionally, the *Navigation bar for construction steps* can be displayed at the bottom of the graphics window, allowing repetition of a construction without giving away the required construction steps ahead of time.

Although GeoGebra's user interface consists of several components, which can be hidden on demand, its design is based on the so called *KISS* principle, known from computer engineering. This principle expresses the goal of a programmer to 'keep it short and simple', in order to maintain the usability of a software [Hohenwarter, 2006b, p. 109]. In the case of GeoGebra, the developer tries to design the user interface of the software in a straightforward and clear way, which supports the model of cognitive processes for learning with multimedia and reduces the cognitive load for the benefit of more successful learning [Clark and Mayer, 2003, p. 38].

3.2.3 Implementation of e-Learning Principles in GeoGebra

The design of GeoGebra's user interface also mirrors the intention of fostering effective learning by considering the e-learning principles stated by Clark and Mayer [Clark and Mayer, 2003].

Multimedia Principle: "Use words and graphics rather than words alone."
[Clark and Mayer, 2003, p. 51]

This e-learning principle is implemented in several ways in GeoGebra's user interface by combining text (in this case numeric and algebraic expressions) with graphical representations [Hohenwarter and Preiner, 2008, p. 7].

At first, the software offers *two views of each object*. The algebraic representation corresponds to the textual component, whereas the graphical representation adds the visual component mentioned in this principle.

Secondly, a *dynamic construction protocol* can be opened and placed next to the graphics window. It contains the name, definition, command, and algebraic expression for each object used in the construction and provides a navigation bar to go through the construction process step-by-step. The current construction step is highlighted within the construction protocol while the corresponding object appears in the graphics window of GeoGebra.

Thirdly, *static and dynamic text* can be inserted into the graphics window to emphasize certain mathematical concepts and relations, show changes in selected algebraic expressions dynamically, highlight mathematical invariants, or carry out calculations.

Finally, the Multimedia Principle also influences the export possibilities of GeoGebra. On the one hand, so called *dynamic worksheets* combine interactive dynamic

figures with explanations and tasks for students. On the other hand, a *construction protocol* can be exported for every construction or dynamic figure giving a textual description of all objects within a table as well as a picture of the actual construction [Hohenwarter, 2006b, p. 110].

Contiguity Principle: “Place corresponding words and graphics near each other.” [Clark and Mayer, 2003, p. 67]

This e-learning principle is also invested in multiple ways within the design of GeoGebra’s user interface by placing corresponding words (here: mathematical expressions) and graphics near each other, making it easier to find corresponding representations of the same object (see [Hohenwarter and Preiner, 2008, p. 7]).

At first, GeoGebra provides *pop up text* that show the definition of an object when the mouse is moved over one of its representations. Additionally, pop up text appears when the pointer hovers over one of the toolbar icons, showing the name of the corresponding tool.

Secondly, *labels of objects* can either consist of the name, the algebraic value, or both the name and value of the object. Since the label follows the movements of its object, the graphical and algebraic representation of the object always stay close to each other.

Thirdly, both representations of an object are displayed in the *same color*, which can easily be modified by the user to distinguish between objects of the same type (e.g. two circles). This makes it easier to find corresponding representations in the algebra window, graphics window, as well as the dynamic construction protocol.

Fourthly, *static and dynamic text* can easily be inserted into the graphics window. They can be placed close to corresponding objects or even attached to them so they follow every movement dynamically.

Coherence Principle: “Adding interesting material can hurt learning.” [Clark and Mayer, 2003, p.111]

This e-learning principle is also taken into account by avoiding unnecessary distractions like glaring colors or decorations within GeoGebra’s user interface [Hohenwarter and Preiner, 2008, p. 7]. Also, unneeded objects can be hidden in both windows to avoid distracting the students and help them to focus on the relevant components of a dynamic figure. In the algebra window, this can be achieved by defining these objects as ‘auxiliary objects’ and hiding them from view, which allows a user to ‘tidy up’ the lists of free and dependant objects. In the graphics window, the appearance of those objects can be either changed so they don’t attract attention any more (e.g. dashed lines, lighter color) or the objects can simply be hidden from view.

3.3 Creating Instructional Materials with GeoGebra

Although GeoGebra was initially developed with the goal of letting students explore and discover mathematical concepts on their own, it turned out to also be a very useful and convenient tool for teacher creation of their own instructional materials. For this purpose, the software offers different export possibilities for dynamic figures, which were designed to be as easy to use as possible, in order to allow a wide range of teachers to realize their own visions of successful instructional materials.

3.3.1 Basic Skills Needed

In order to use GeoGebra for creating instructional materials, teachers need to master a certain range of basic computer skills, including the use of text processing software (e.g. MS Word, Writer[CollabNet Inc., 2008c]) or presentation software (e.g. MS PowerPoint, Impress[CollabNet Inc., 2008b]). Although knowledge about the basic use of GeoGebra is essential, experience from in-service teacher workshops shows that the main problems concerning the creation of instructional materials for secondary school mathematics often derive from a lack of expertise when working with computers (see model of computer competencies in [Fuchs and Landerer, 2005]).

The following list of basic computer skills come in handy for teachers who are creating their own instructional materials and want to give them to their students in digital form. Depending on the type of instructional material, not all of these skills are necessary, but they definitely provide an advantage if mastered by the teachers.

File handling: Basic skills concerning the handling and organization of files and folders.

- Teachers need to be able to create and name a new folder.
- Teachers need to know how to save files in different programs.
- Teachers need to understand the extensions of the file names (e.g. *‘.ggb’* for a GeoGebra file) in order to identify them and be able to handle them using appropriate software.
- Teachers need to be able to navigate within the folder structure of their computers.

Picture file handling: Basic skills concerning the use of images, which can be used to enhance instructional materials.

- Teachers need to be able to identify image files by means of file name extensions.
- Teachers should know how to resize an image using appropriate software (e.g. Irfan View).
- Teachers should know how to find and download image files from the Internet and be aware of copyright issues.

- Teachers should know about the ‘resolution’ of images necessary for different purposes (e.g. printout, presentation).

Text processing software: Basic skills necessary to create instructional materials using text processing.

- Teachers need to be familiar with the basic use of a text processing software.
- Teachers should know how to create tables in order to enhance the layout of their instructional materials, as well as how to apply basic formatting to the text.
- Teachers should know how to use an equation editor in order to create appealing materials for mathematics teaching and learning.
- Teachers need to know how to insert an image into a text processing document.
- Teachers need to know how to resize an image within a text processing document.
- Teachers need to know how to check if a picture is displayed in original size in order to maintain its scale on a printout.
- Teachers need to know how to print a file.

Accessibility and Internet: Basic skills necessary to deploy instructional materials other than handing out paper copies.

- Teachers need to know how to distribute files using CDs or USB drives.
- Teachers need to know how to copy and paste files and folders from a storage device to a computer.
- Teachers should know how to upload files to an Internet server in order to provide them to students online.
- Teachers should be familiar with basic handling of a webpage, including the creation of hyperlinks in order to make materials accessible.

3.3.2 Static Instructional Materials

GeoGebra supports the creation of static instructional materials, such as handouts, worksheets, tests, or presentations, which can be printed out and literally handed to the students. The following options are available in GeoGebra.

Printing a GeoGebra construction: Each GeoGebra construction can be printed on paper. After selecting the corresponding menu item, a dialog window appears allowing the user to specify title, author, and date of the construction. A print preview is provided and the scale of the graphic can be set by the user.

Printing the *Construction protocol*: After opening the *Construction protocol*, its print preview can be selected from the menu of the resulting dialog window. Again, title, author, and date can be specified before printing the construction steps.

Exporting the drawing pad to the clipboard: Using menu items the entire drawing pad or actual selection can be exported to the computer's clipboard. Afterwards, it can be inserted as an image into a word processing or presentation software. The image is not saved as a picture file and thus, can't be accessed later on. This export option represents a quick way to create sketches and constructions for handouts or worksheets and requires minimum knowledge about handling and inserting picture files into a word processing or presentation software. Nevertheless, the scale of the graphic can't be modified when using this export option in GeoGebra.

Exporting the drawing pad as a picture: This export option can be accessed over the menu, which opens a dialog window that allows user specification of the picture format, as well as scale and resolution of the image file. Additionally, the printing size of the picture is displayed, allowing users to determine if the image can be expected to fit on a sheet of paper without resizing it. Afterwards, the image file needs to be saved in order to be able to subsequently insert it into a word processing or presentation software. The insertion process requires navigating through the folder structure of the computer in order to find the saved picture file.

3.3.3 Interactive Instructional Materials

GeoGebra allows creation of web-based interactive instructional materials, so called *dynamic worksheets* [Hohenwarter and Preiner, 2007a, Hohenwarter, 2006b] which are also referred to as *mathlets* or *virtual manipulatives*. These interactive materials can be used both on local computers or via the Internet and require some kind of browser software installed on the computer, as well as Java 1.4.2 or later which can be downloaded from the Internet for free² as necessary. Students don't need to know anything about the use of GeoGebra in order to work with these materials and don't need GeoGebra installed on their computers. Since dynamic worksheets can also be provided online, students can use them both in school and at home.

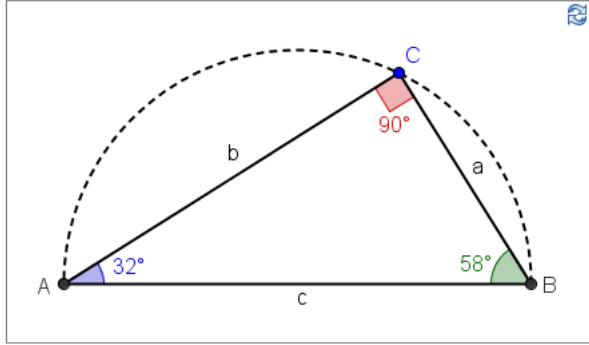
Dynamic worksheets are webpages that usually consist of a dynamic figure (in case of GeoGebra, an interactive Java-applet) and corresponding explanations, as well as questions and tasks for students. They can contribute to a better understanding of mathematical concepts by allowing for interactive manipulations of the provided dynamic figure, and can foster active learning, as well as mathematical experiments. Additionally, dynamic worksheets can support guided discovery learning and encourage self-dependent learning as well as mathematical inquiries [Bruner, 1961, Joolingen, 1999].

Figure 3.2 shows a dynamic worksheet that allows students to explore the theorem of Thales. Vertex C of the triangle ABC lies on a semicircle over segment \overline{AB} . Following

²Java download: www.java.com

Theorem of Thales

Below you can see a triangle whose vertex C lies on a semicircle over segment c .



1. Drag point C with the mouse along the semicircle and watch the angles of the triangle. Write down your observations on a sheet of paper.
2. Try to phrase a conjecture including the words *semicircle*, *triangle*, and *angle*.
3. Check your conjecture for at least 3 different triangles.
Hint: Drag points A and B with the mouse to create a new triangle.

Figure 3.2: Example for a dynamic worksheet

the instructions below the dynamic figure, students are guided towards discovering the meaning of this theorem, stating that such a triangle is always a right triangle. After coming up with a conjecture, students can verify it for a number of different triangles, which can be created by moving points A and B with the mouse to another position.

Creating Dynamic Worksheets

The creation process of dynamic worksheets is rather straightforward in GeoGebra. Depending on the visual status of the algebra window (shown or hidden), either both the algebra and the graphics window, or just the graphics window are displayed within the interactive applet. After creating an appropriate dynamic figure, an export dialog can be opened using the menu *File – Export – Dynamic Worksheet as Webpage (html)...*. This dialog allows the definition of a title for the dynamic worksheet, as well as entry of both the author's name and the date in the upper part of the dialog window.

By default, the *General* tab is activated, which provides two text fields, enabling the user to enter text that should be displayed above and below the dynamic figure. Additionally, the user can choose if the interactive applet should be displayed within the dynamic worksheet, or if a button should be included, allowing students to open the GeoGebra application window in order to access the dynamic figure [Hohenwarter and Preiner, 2007a, p. 4 – 6].

The dynamic worksheet can be saved after filling in the text fields and clicking the 'Export' button. Since, at this time, several files are saved on the computer, the creation of a new folder is recommended before exporting the dynamic worksheet. Thereby, all

created files needed for dynamic worksheet functionality will stay together in one folder. This is especially important when making the interactive worksheet available for students who will only need to open the file with the extension ‘html’ in any Internet browser [Hohenwarter and Preiner, 2007a, p. 8].

Enhancing Interactive Instructional Materials

GeoGebra additionally allows for further enhancement of dynamic worksheets and additions to their interactivity. The enhanced features can easily be accessed using the *Advanced* tab of the export dialog for dynamic worksheets [Hohenwarter and Preiner, 2007a, p. 23 – 24].

Functionality: Users can determine...

- if students should have access to the *Context menu* by right-clicking (MacOS: *Ctrl*-clicking) on the interactive applet, providing them a wider range of modification possibilities.
- if a reset icon should be displayed in the upper right corner of the interactive applet, allowing students to reset the dynamic figure to its initial state.
- if double clicking on the interactive applet should open the GeoGebra application window in order to allow further investigation of the dynamic figure.

User Interface: Authors can customize the interface of the interactive applet by displaying...

- the menubar which allows students, for example, to save or print their work, or to display coordinate axes on the drawing pad.
- the toolbar which gives students access to the dynamic geometry tools of GeoGebra. The number of tools available to the students can be reduced by customizing the toolbar prior to exporting the dynamic worksheet [Hohenwarter and Preiner, 2007a, p. 24].
- the toolbar help which can only be shown in combination with the toolbar and provides information on how to use the dynamic geometry tools.
- the input field of GeoGebra which enables students to enter algebraic expressions.

Additionally, the width and height of the interactive applet can be modified.

Java Applet: When the dynamic worksheet is exported, the minimum number of required files is created and saved on the computer. Whenever the dynamic worksheet is opened, the missing information is provided by the GeoGebra web server. This option has the advantage that the user has to deal with less different files, but requires an Internet connection in order to maintain the functionality of the dynamic worksheet.

Creating Interactive Exercises

For advanced users, who already have some experience with HTML and programming, GeoGebra offers a JavaScript interface for the interactive applets of dynamic worksheets. By this means, additional elements, such as text fields or buttons, can be inserted into the web page, allowing for increased interactivity of the dynamic worksheet. It is possible to create interactive exercises that check the student's answer to a problem and give feedback about their progress [Hohenwarter and Preiner, 2007a, p. 26 – 27].

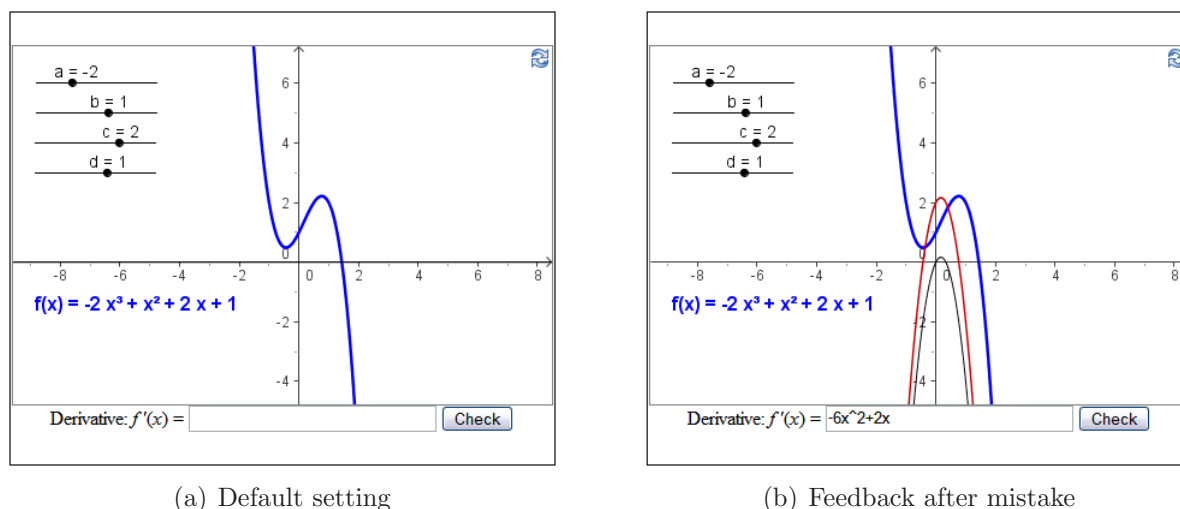


Figure 3.3: Example for an interactive exercise

Figure 3.3 shows two screenshots of such an interactive exercise. Within the applet, the equation and graph of a cubic polynomial are displayed, whose parameters can be modified by using the sliders shown in the upper left corner. Below the applet, an input field was inserted, allowing students to enter the derivative of the displayed polynomial (see figure 3.3(a)). After clicking the 'Check' button, students get feedback about their solution. If the solution is not correct, both, the graphs of the correct derivative, as well as the graph corresponding to the student's solution are displayed and students are encouraged to compare the graphs and correct their mistake (see figure 3.3(b)).

3.4 Teaching Mathematics with GeoGebra

Since GeoGebra combines the ease of use as well as the construction features of a dynamic geometry software with the power and functionality of a computer algebra system, it opens up a wide range of application possibilities for teaching mathematics. Its versatility enables teachers to use the software in all grade levels from secondary school up to college and for a wide range of different mathematical topics. Accordingly, GeoGebra can be used as a presentation tool as well as for the creation of instructional materials, such as notes

or interactive worksheets [Hohenwarter, 2005, Fuchs and Hohenwarter, 2005]. Since the software initially was developed for the use by students, it fosters active and discovery learning [Bruner, 1961], and can easily be used by students to carry out mathematical experiments.

3.4.1 GeoGebra as a Presentation Tool

No matter how user friendly an educational software is designed to be, a certain amount of time needs to be spent in order to get to know the software and to become comfortable enough to actually use it for everyday teaching. On the one hand, a teacher needs to know about the basic functionality of software like GeoGebra and needs to reach a certain expertise using this educational tool. On the other hand, a teacher also needs to know about teaching methods that allow a successful integration of dynamic mathematics software into everyday teaching.

Static Visualizations

Experience in teacher workshop suggests that most teachers who are introduced to GeoGebra tend to use it as a presentation and visualization tool at first. They usually prepare instructional materials at home so they don't have to actually operate the software in front of their students. Classroom presentation competencies seem to require a higher level of confidence and tend to evolve after some practice time using the software.

As a first approach, many teachers start to use GeoGebra in order to create sketches and constructions for presentations, handouts, notes, or quizzes. Instead of using the software during their classes, teachers can take their time to create the materials they need. This phase of indirectly integrating GeoGebra into their teaching allows them to practice using the software while exploring its versatile application possibilities without the pressure of taking care of their students at the same time. This approach of integrating educational software into 'traditional' teaching [Fuchs and Hohenwarter, 2005, p. 130] requires a minimum of technical equipment in the classroom and therefore, could be implemented by almost every teacher who is willing to enhance their everyday teaching of mathematics.

Dynamic Visualizations

As they gain more confidence in operating the software, teachers can enter the next phase of integrating GeoGebra into their everyday teaching. They prepare their own construction files and dynamic figures that can be used for presentation purposes and dynamic visualization of mathematical concepts. In this phase, teachers begin to use the dynamic and interactive functionality of GeoGebra allowing their students to benefit from dynamic visualizations. Additionally, teachers tend to notice that the software is capable of facilitating their everyday teaching since a lot of students can understand concepts better if they can see how objects are related and changed dynamically.

For example, several teachers observed in Florida like to use GeoGebra in order to visualize the impact of geometric transformations such as reflections or translations. Using a dynamic visualization to explain the concept, students tend to grasp the mathematical ideas easier than with traditional teaching methods which involve graphing transformations of polygons point-by-point and memorizing keywords like ‘flip’ for reflections or ‘slide’ for translations.

In this phase of integrating GeoGebra into teaching, teachers need access to a computer and projector in their classrooms in order to actively use GeoGebra for presentation purposes. Teachers don’t necessarily have to do a construction in real-time and from scratch, but could also use the *Construction protocol* and *Navigation bar* in order to review a construction step-by-step.

Once teachers get comfortable with using prepared GeoGebra materials during their classes, the transition to actually creating geometric constructions and other dynamic figures in real-time and from scratch can easily be made. In this phase, teachers begin to treat GeoGebra like every other teaching tool, a tool that can easily be integrated into everyday teaching without necessarily having to prepare special materials ahead of time. This not only increases the flexibility of teachers in terms of reacting to students’ questions or conjectures, but also allows for an extension of the range of teaching methods. For example, teachers can carry out spontaneous mathematical experiments in order to explain certain mathematical concepts, or they can encourage so called “What if...” questions [Edwards and Jones, 2006, p. 30], which could not be easily answered and visualized with static constructions and sketches.

3.4.2 Best Practice Examples for Everyday Teaching

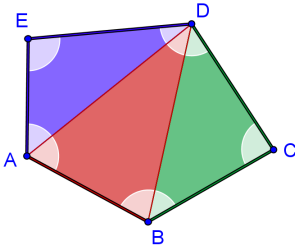
Finding the Sum of Interior Angles of a Polygon

Figure 3.4 shows a ‘traditional’ worksheet that guides students towards finding a formula to calculate the sum of interior angles of a polygon. The sketch of the pentagon was created with GeoGebra and exported as a picture before inserting it into text processing document (see section 3.3.2).

After calculating the actual angle sum of the pentagon shown on the worksheet by subdividing the polygon into several triangles, students are encouraged to draw some additional polygons with different numbers of vertices. By filling in the missing values of the displayed table, they should be able to find patterns within the recorded values for number of vertices, number of sides, number of possible triangles, and sum of interior angles for each of the polygons. Finally, students should try to ‘translate’ their findings into a mathematical formula, that allows them to calculate the sum of interior angles for an arbitrary polygon with n vertices.

Depending on the age, mathematical knowledge, and skills of students, this worksheet could be integrated into a ‘traditional’ teaching situation in several different ways.

Calculating the sum of interior angles of a polygon



1. Try to **find the sum** of interior angles of the pentagon $ABCDE$.
Hint: What is the angle sum of a triangle?
2. **Explain** your answer and write it down.
3. Use the **table** below and **fill in the missing values** for polygons with 3, 4, 5, and 6 vertices.
Hint: You might want to draw sketches of the corresponding polygons.
4. Come up with a **conjecture** about calculating the sum of interior angles for any polygon with n vertices.
Hint: Look for patterns in each column!
5. Fill in the last row of the table by translating your conjecture into a mathematical **formula**!

Vertices	Sides	Triangles	Angle sum
3	3	1	180°
4		2	360°
5	5	3	
6			
n			

Figure 3.4: Worksheet: Sum of Interior Angles of a Polygon

Teacher centered approach: After distributing the copies of the worksheet, the teacher could guide the students through the tasks. Students could work along with the teacher while every step of the process should be discussed and explained thoroughly.

Student centered approach: Students work either alone or in pairs and try to figure out the formula on their own. The teacher could provide another worksheet with sketches of different polygons facilitating the process of filling in the missing values of the table. Students who finish early could be encouraged to help their colleagues. Once all (or most of the) student teams find a solution for the problem, their answers could be discussed with the teacher making sure that every student understands the concept behind the formula.

Mixed approach: After a short introduction and discussion of the angle sum of a triangle, students could be encouraged to work on tasks 1 through 3, either alone or in pairs.

After filling in the missing values of the table, the teacher could take over and guide them towards finding patterns within the table columns, helping them to find a general formula for the sum of interior angles of an arbitrary polygon.

Integer Addition on the Number Line

Figure 3.5 shows two stages of a dynamic visualization of integer addition on the number line. Sliders a and b can be moved with the mouse in order to create a new addition problem. The corresponding numbers are displayed as arrows on the number line and automatically adapt to the modifications of the sliders. Additionally, the actual addition problem is displayed in the right upper corner of the dynamic figure.

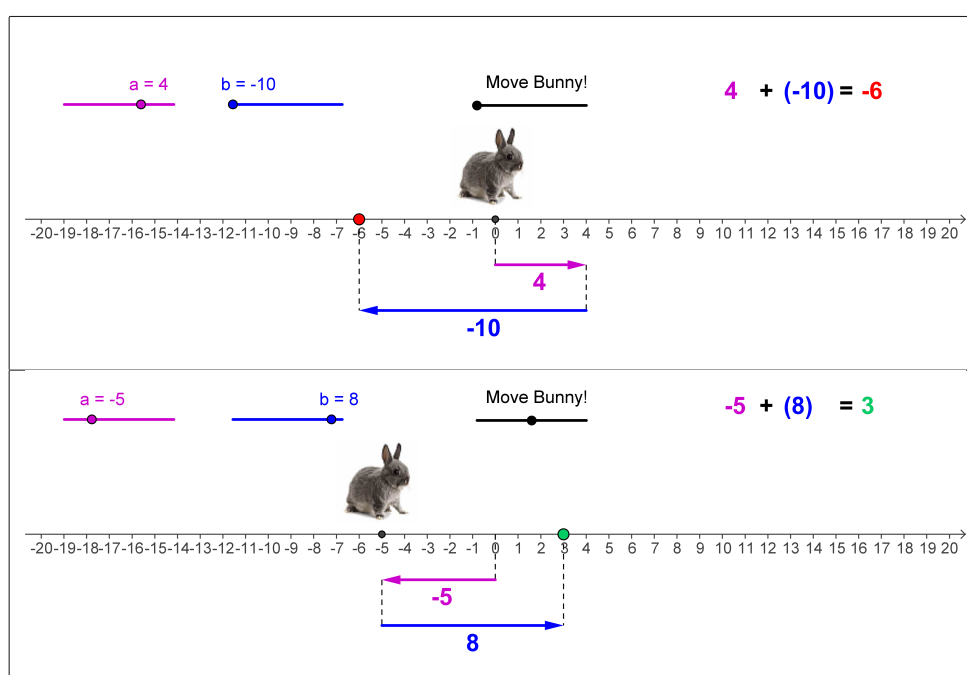


Figure 3.5: Dynamic visualization: Integer addition on the number line

After creating a new addition problem, slider *Move Bunny!* can be dragged with the mouse causing the bunny to move along the number line. Starting at 0, the bunny moves according to the sign of slider a 's value, turns around once it reaches the corresponding value on the number line (if necessary), and continues corresponding to the value of slider b , ending at the position of the solution to the addition problem.

Since this dynamic figure is intended to be used by the teacher, it doesn't contain any instructions for students. In order not to confuse students, it was exported into a webpage and it doesn't display all portions of GeoGebra's user interface. This applet could be used to introduce the concept of integer addition on the number line or to check students' answers to addition problems by dynamically visualizing the process using the picture of the bunny.

Introducing the Concept of Slope Function

Figure 3.6 shows a screen shot of a dynamic visualization of the slope function of a given cubic polynomial. Sliders a , b , c , and d can be used to modify the parameters of the polynomial function and thus, to create a number of new polynomials. Point A is restricted to move along the polynomial and can be dragged with the mouse. Additionally, tangent t through point A and its slope triangle are displayed, as well as a special point S , both following and adapting to the movements of point A . Point S has the same x -coordinate as point A and the value of the tangent's slope as its y -coordinate.

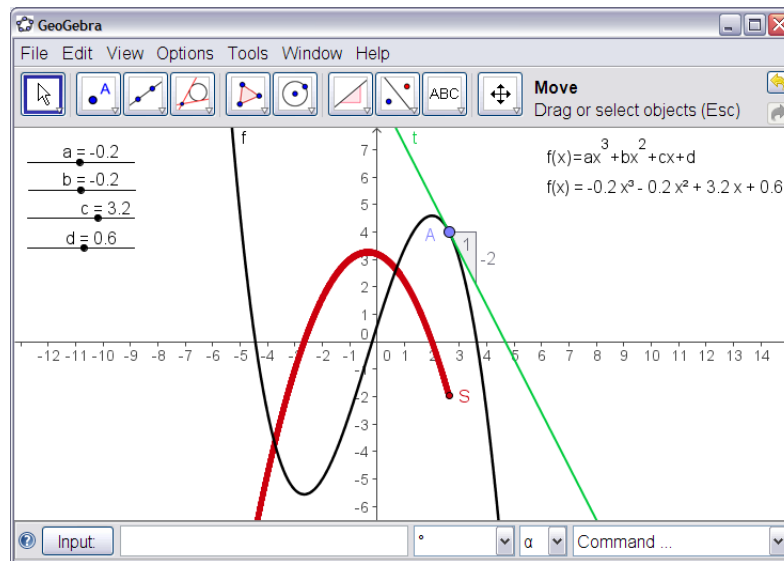


Figure 3.6: Dynamic visualization: Slope function

This dynamic visualization is intended to introduce students to the concept of a slope function and could be used by the teacher to visualize how the graph of the slope function is related to the slope of the tangent line for each point on the polynomial. Students can formulate conjectures about the path of point S and their conjectures can then be checked by turning on the trace of this point. After categorizing the graph of the slope function, students could try to find the corresponding equation. Using the input field of GeoGebra, their results can be entered and verified using the dynamic figure.

In order to generalize the students' conjectures, new polynomials can be created using the sliders, allowing inspection of a number of different polygons. Once the students grasp the concept of a slope function, the teacher can introduce the term 'Derivative'. By systematically recording the equations of a series of 'special' polygons (e.g. $f_1(x) = x^2$, $f_2(x) = 2x^3$, $f_3(x) = x^4 + 2x, \dots$) and their slope functions (e.g. $s_1(x) = 2x$, $s_2(x) = 6x^2$, $s_3(x) = 4x^3 + 2, \dots$), students could also be guided towards finding a pattern and therefore, discovering rules to calculate derivatives of polynomials by themselves.

Furthermore, this dynamic figure allows function f to be redefined in order to obtain other types of functions (for example trigonometric functions) whose slope function and derivatives could be explored in the same way.

3.4.3 Discovery Learning and Lesson Enrichment

Once teachers feel comfortable with using GeoGebra for presentation purposes and feel self-confident about operating the software during everyday teaching situations as well as solving problems that might occur, they could also begin to let their students actively discover mathematical concepts on their own. ‘New’ teaching methods, such as mathematical experiments and discovery learning [Bruner, 1961], could be combined with ‘traditional’ teaching methods in order to create a learning environment that fosters active and student centered learning [Fuchs, 1990].

No matter if students explore mathematics alone or in groups, the teacher should try to be an advisor in the background who gives support when help is needed. The students’ results of their experiments with *GeoGebra* should be the basis for discussions in class. This gives teachers more time to concentrate on fundamental ideas [Schweiger, 1992] and mathematical reasoning. [Fuchs and Hohenwarter, 2005, p. 130]

Not only can teaching methods be influenced by the use of dynamic mathematics software, but also the type of problems, and level of mathematical concepts covered. In fact, the general attitude of students towards mathematics itself could potentially be changed.

Perhaps utilising *GeoGebra* could inspire a change from regular forms of enrichment / extension activity to things that need *high level thinking*, and things that pupils may find themselves *wanting to follow-up* outside school lessons. [Edwards and Jones, 2006, p. 30]

Depending on the computer literacy of students and their expertise concerning the use of GeoGebra, different ways of integrating the software into their learning environment can be implemented.

No Direct Contact with GeoGebra

Once teachers have mastered the basic use of GeoGebra and gained enough self-confidence and experience to use the software for teaching mathematics, they often wish that their students could benefit from using the software as well. But knowing about the time and effort they had to invest in order to become proficient, they often are reluctant to repeat this whole process with their students. Since teaching time always seems to be too short and limited to cover mandatory topics, a lot of teachers can’t afford to take the time in order to introduce their students to GeoGebra.

During the last few years, this scenario was observed with Austrian as well as Floridian teachers, who additionally need to constantly be aware of the next standardized testing phase and therefore, need to stick to the curriculum as closely as possible. In order to help teachers let their students benefit from mathematical enquiries and discovery learning [Bruner, 1961], GeoGebra offers the possibility to export dynamic figures as web pages, so called dynamic worksheets, which can be provided to the students locally on their computers, over the school network, or even using the Internet (see section 3.3.3).

Students don't need to know anything about the use of GeoGebra and only need basic computer skills in order to work with such prepared interactive materials. Thus, dynamic worksheets can effectively be used without any formal introduction to GeoGebra, and are available both at school and at home where they can foster mathematical experiments and discovery learning.

Restricted Use of GeoGebra

In order to increase the interactivity of dynamic worksheets, as well as to allow the students more freedom concerning the exploration of mathematical concepts, GeoGebra allows users to customize the toolbar in order to limit the number of available tools. This feature is especially useful for teachers who only want to introduce a selection of dynamic geometry tools at a time and limit the amount of technical information that needs to be given to the students.

On the one hand, teachers could prepare a dynamic worksheet, which provides a customized toolbar with a selection of tools necessary to explore the given dynamic figure along with the toolbar help in order to assist students to find out how to operate these tools. In this way students don't need to have GeoGebra installed on their computers. The teacher also could include tasks and explanations on the dynamic worksheet that guide students towards discovering the mathematical concept visualized in the dynamic figure.

On the other hand, teachers might provide a 'usual' GeoGebra file to the students, which contains a customized toolbar and therefore limited functionality. In this case, students finally have direct contact with the software itself, which could be a first step towards the independent use of GeoGebra. In this scenario students need to have GeoGebra installed on their computers in order to be able to open the prepared GeoGebra file. Additionally, tasks and instructions need to be given in a separate document, perhaps as part of a more traditional paper worksheet, in order to guide the students' experiments into the desired direction.

Independent Use of GeoGebra

Independent use of GeoGebra by students should be the final goal of every teacher who starts to use the software for teaching mathematics. By introducing students to the full functionality and potential of GeoGebra, teachers provide them with a valuable and versatile tool with the potential to influence and change their general attitude toward ma-

thematics and foster their understanding of connections between different mathematical disciplines, like geometry, algebra, and even calculus.

Considering the potential positive impact of GeoGebra on students' mathematical skills if used for a longer period of time, ideally throughout several grade levels, a thorough introduction of GeoGebra would most likely pay eventual dividends. The software could be introduced in the lower grade levels of middle school in order to build a basis for an extensive use when dealing with more complex mathematical topics later on in high school. In Austria, since teachers have opportunity to teach the same students in different grade levels (theoretically starting in 5th grade and up to 12th grade), this concept could be implemented rather easily compared to the situation in Florida. In Florida, teachers usually teach a specific grade level, or even a certain mathematics course, keeping them from being able to work with the same students for more than one year. In this case, cooperation between different mathematics teachers needs to be strengthened in order to allow students to benefit from GeoGebra throughout a longer period of time.

In order to introduce students to GeoGebra and facilitate their first contact with the software, different approaches are possible. On the one hand, teachers could introduce the software step-by-step, starting with the use of basic dynamic worksheets and increasing the knowledge about using the software slowly by introducing more and more tools and features. On the other hand, students could also be exposed to the full functionality of GeoGebra right away by using step-by-step instructions as well as different teaching methods (e.g. work along with the teacher, work in groups or on their own). The focus should be kept on selected tools that are useful for a certain mathematical concept. Furthermore, students could also be encouraged to explore GeoGebra on their own, for example by drawing colorful pictures, which is already possible for students in early middle school.

No matter which approach is chosen, students never should be overwhelmed with technical information and be given all the support necessary to help them feel comfortable using a software like GeoGebra to explore mathematics. Thereby, mathematical concepts and topics should always take the center stage, while the software should just represent a useful tool to increase the mathematical understanding and foster students' learning.

3.4.4 Best Practice Examples for Student Centered Teaching

Parameters of a Linear Equation

Figure 3.7 shows a screenshot of a dynamic worksheet that allows students to explore the connection between the parameters of a linear equation and its graphical representation as a line. In order to successfully use this interactive worksheet, students don't have to know anything about the use of GeoGebra and only need minimal computer literacy to explore the mathematical concept visualized in the dynamic figure.

Sliders m and b can be dragged with the mouse in order to modify the position and direction of the line, encouraging students to come up with a conjecture about how the line can be affected. Using checkbox 'Equation', students can display the corresponding linear equation in order to examine how the slider values are related to the parameters

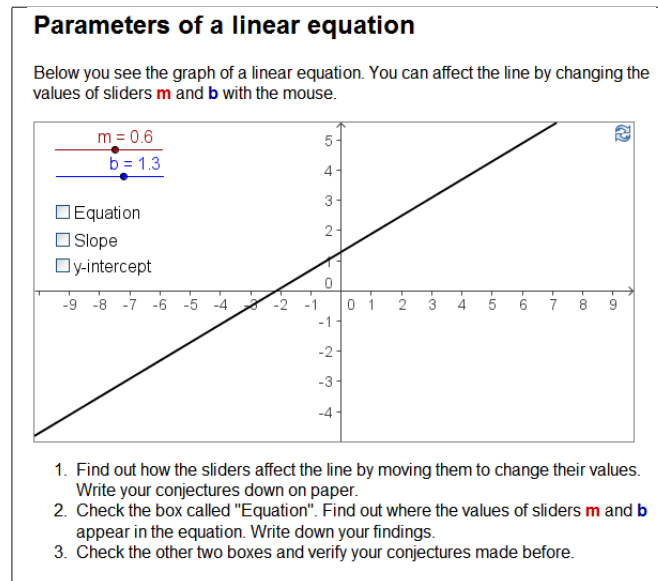


Figure 3.7: Dynamic worksheet: Parameters of a linear equation

of the linear equation. Finally, the other two checkboxes can be used to check students' observations by displaying a slope triangle, as well as the distance of the y -intercept to the origin of the coordinate system.

Working on this dynamic worksheets either alone or with a colleague, students are guided towards discovering the connections between the graphical and algebraic representation of a linear equation. This dynamic worksheet could be used as a follow-up activity after a general introduction of linear equations to students. Students could work on this activity either in class or at home, following the instructions below the dynamic figure. Afterwards, their findings should be discussed in class in order to make sure that they really were able to grasp the connection between the different representations of a linear equation.

Constructing Tangents to a Circle

Figure 3.8 shows another dynamic worksheet that allows students to find out how to construct tangents to a circle and redo the construction process on their own.

The toolbar shown in the applet on the right hand side was customized in order to limit the number of available tools and, thus, to facilitate the construction process for the students. Additionally, the toolbar help is displayed, allowing students to find out how to use each of the tools available.

This dynamic worksheet consists of two dynamic figures as well as tasks to guide students through constructing tangents to a circle. The applet on the left hand side contains a navigation bar, allowing step-by-step review of the construction. After familiarizing themselves with the construction process, students can use the applet on the right hand side in

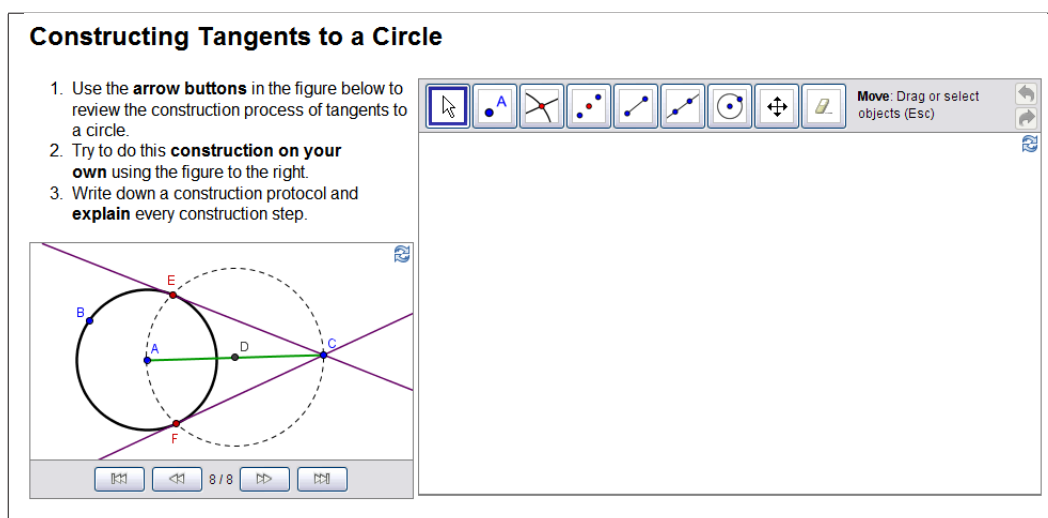


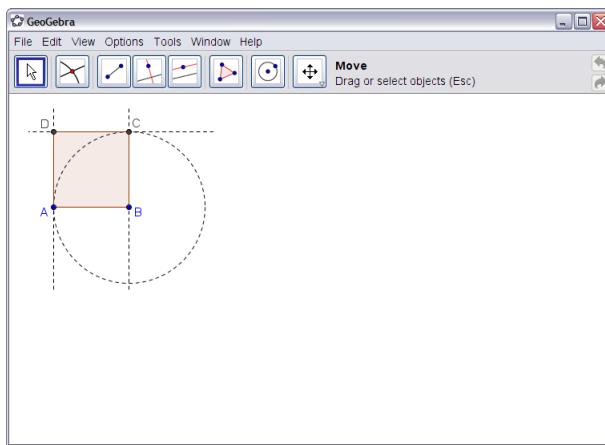
Figure 3.8: Dynamic worksheet: Constructing tangents to a circle

order to redo the construction using the provided tools. They can switch between the two applets at any time in order to check their progress and the correctness of their construction. In a final step, students are encouraged to write down the construction process in their own words which serves as a basis for the discussion of their results in class later on.

Constructing a Square

Figure 3.9(a) shows a screen shot of a prepared GeoGebra file with a customized toolbar that only consists of those tools necessary to construct a square. In order to facilitate this process, the sketch of a possible square construction is shown in the background of the drawing pad. Since the number of tools available is restricted, their functionality could be discussed with the students before they actually begin to work on the task in order to make sure that everyone knows how to operate the tools.

Using this prepared GeoGebra file instead of the full version of GeoGebra, the possibility that students will be overwhelmed is reduced. Instead, student focus is more pointedly directed to those tools that are necessary to create the corresponding construction. An additional worksheet should be provided to the students, containing instructions on how to work with the prepared file and how to solve the given task (see figure 3.9(b)). Furthermore, students could be encouraged to try and find different ways of constructing a square. Afterwards, students' solutions should be discussed in class, giving them the chance to explain their results, as well as to talk about different ways of constructing a square and explain why they work and which geometric properties are involved.



(a) Prepared GeoGebra file

Constructing a square

1. Summarize the **geometric properties** of a square.
2. Open the file `square_construction.ggb` in **GeoGebra**.
3. Try to **redo the square construction** shown in the sketch. **Explain** every construction step on paper.
4. Apply the **drag-test** to your square in order to check if you construction is correct.
5. **Bonus task:** Try to find at least one **other way of constructing a square**. Write down a **construction protocol** for your own construction process.

(b) Additional tasks

Figure 3.9: Constructing a square

Chapter 4

Introductory Workshops for GeoGebra

In order to identify difficulties that occur during the introduction process of dynamic mathematics software to novices, a series of four introductory workshops for GeoGebra was evaluated during an NSF MSP summer institute in July / August 2007. Participating secondary school teachers filled in several surveys and questionnaires giving feedback about the design of the workshops as well as about the usability of the software GeoGebra. Afterwards, the quantitative and qualitative data collected was statistically analyzed and the results were interpreted based on prior experience with introducing the software. Eventually, the findings of this study will provide a basis for the development of potentially more successful introductory workshop and accompanying materials for dynamic mathematics software in general, and GeoGebra in particular.

In this chapter the nature and methodology of the research study is described. At first, the context of the study as well as the workshop design, objectives, structure, content, and teaching methods are summarized. Secondly, the methodology of the evaluation process is described and the research questions of the study are stated. Finally, the evaluation instruments and the statistical tests used to analyze the collected data are described.

4.1 Context and Environment

4.1.1 Math and Science Partnership Project

Based on ten years of prior teacher enhancement projects, the Math and Sciences Partnership (MSP) project between the Department of Mathematical Sciences at Florida Atlantic University (FAU) and the School Board of Broward County (SBBC) started in August 2004 funded by a grant from the US National Science Foundation (NSF).

The principal investigators of this five year program called *Standards Mapped Graduate Education and Mentoring*, Dr. Heinz-Otto Peitgen and Dr. Richard F. Voss, designed a special curriculum that consists of advanced mathematical content as well as best prac-

tice examples for the use of technology in teaching mathematics. The program's curriculum is designed to meet the needs of middle school mathematics teachers and to provide mathematical and technological content connected to their everyday teaching situation in classrooms. Through evening classes, annual summer institutes, as well as biannual pedagogy conferences the participating in-service teachers are able to earn a Master's degree in Science and Teaching at FAU. The three major goals of this project are to . . .

- increase mathematical and pedagogical content knowledge,
- effectively use technology for learning and teaching mathematics, and
- impact university as well as school district level student performance.

In August 2006 and May 2007, two supplemental grants were approved by NSF that enabled an intensified integration of the dynamic mathematics software GeoGebra into the project. This addition now reinforces the technology component of the project in several ways:

Best Practice Examples: Experienced teachers present best practice examples of their use of GeoGebra and other technology in the classroom and discuss their experiences with other teachers. The selective use of technology is reinforced in order to foster an effective use of technology in the classroom.

Multiple Representations: GeoGebra provides both an algebra and a graphics window to represent mathematical objects. This supports the use of multiple representations in course journals and materials created by the participating teachers.

Discovery learning: Teachers create their own interactive instructional materials using appropriate software in order to foster student-centered and discovery learning [Bruner, 1961, Joolingen, 1999] in their classrooms.

Collaboration: Teachers in the project form a professional community that is connected to teachers from around the world using collaborative software such as wikis and forums on the Internet.

4.1.2 MSP Summer Institute 2007

An important component of this MSP project in Florida is the annual summer institute that usually takes place in July / August and lasts for three weeks. During the first week of the institute, teachers who are already taking the graduate classes of the MSP project, prepare technology enhanced mathematics workshops. Those workshops are hosted during the second and third week of the institute for a number of 'new' in-service teachers who haven't been involved in the MSP project so far. The participants are exposed to mathematical content as well as to the use of technology for teaching and learning mathematics, and they experience best practice examples on how to transfer their new skills and knowledge into their classrooms.

In addition to the workshops, several lectures are given by professors and by the most advanced teachers and graduates from the MSP program in order to present more advanced mathematical topics and increase the participants' interest in exploring so far 'unknown' mathematical concepts. Furthermore, the summer institute provides a basis for the recruitment of in-service teachers who want to take part in the graduate classes of the MSP program in order to get a Master's degree in Science and Teaching.

In 2007, the NSF MSP summer institute was dedicated to introduce the participating in-service teachers to a selection of software programs for teaching and learning mathematics (GeoGebra, MS Excel, MS PowerPoint, and MS Word) in order to prepare potential graduate students for the use of technology in the upcoming MSP graduate classes. The participants experienced an intensive introduction to the dynamic mathematics software GeoGebra which consisted of a series of four introductory workshops of 70 minutes each on four consecutive days of the summer institute. The 44 participating secondary school teachers were divided into three groups whereby each group passed through the same sequence of GeoGebra introductory workshops. All workshops were hosted by Dr. Markus Hohenwarter, the main developer of GeoGebra, who not only is professional in using the software, but also has a lot of experience with introducing GeoGebra to newcomers as well as to advanced users. Additionally, several helpers were assigned to each of the three groups in order to support the presenter and assist the participants.

4.1.3 Reasons for Selecting GeoGebra

Although to date there is little research related to the effective integration of GeoGebra into teaching and learning mathematics, there are several reasons for selecting this software to be an essential element of the suggested technology professional development for mathematics teachers.

Open source: GeoGebra is open source software and thus free for non-commercial use. Since teachers and students can download the software from the Internet, it can be used in school as well as at home without any limitations [Hohenwarter and Lavicza, 2007, p. 51]. GeoGebra also offers a so called *WebStart* version allowing users to launch the software directly from the Internet. No special permissions are required from the user in order to install the software on a computer. Thus, GeoGebra can be easily installed in computer labs or on students' home computers. Another advantage of GeoGebra WebStart is that updates are made automatically, guaranteeing the user will always have access to the newest version of GeoGebra.

DGS is effective: Research on dynamic geometry software like *Cabri Geometry* indicates (i) that this kind of software can be effectively integrated into mathematics teaching and learning and (ii) that it has the potential to foster student-centered, active learning [Sträßer, 2001, Laborde, 2001, Erez and Yerushalmy, 2006, Sträßer, 2002, Jones, 1999]. Since GeoGebra's dynamic geometry component is very similar to

Cabri Geometry, the assumption was made that these findings are also true for the use of GeoGebra. Furthermore, “[t]he unanticipated success of GeoGebra has demonstrated that non-commercial software packages have the potential to impact mathematics teaching and learning worldwide.” [Hohenwarter and Lavicza, 2007, p. 51]

Dynamic Mathematic Software: GeoGebra is dynamic mathematics software that combines the ease of use of dynamic geometry software with selected features of a computer algebra system. Future development plans for GeoGebra include the integrated addition of a computer algebra system user interface as well as a spreadsheet component. Both of these will be dynamically connected to the already existing dynamic geometry and numeric algebra component. As a result, GeoGebra promises to become an even more versatile and powerful tool for mathematics teaching and learning [Hohenwarter and Jones, 2007, p. 130].

Multiple representations: GeoGebra provides multiple representations of mathematical objects which potentially foster students’ understanding of mathematical concepts [Duval, 1999]. The numerical algebraic representations of objects are displayed either in the algebra window, as labels of objects, or within text fields directly on the drawing pad. Additionally, the graphical representations of those objects are displayed in the graphics window, and a textual description is shown in the interactive construction protocol. In the future, the set of mathematical representations will be extended by a symbolic algebra as well as a spreadsheet component.

Bidirectional connection: In GeoGebra the need for a bidirectional combination of dynamic geometry and computer algebra [Schumann, 1991, Schumann and Green, 2000] has been realized. The different representations of mathematical objects are dynamically connected enabling GeoGebra to adapt each representation to modifications of its counterpart.

Designed for students: One objective of GeoGebra’s development was to create a software that can be used by students of different grade levels. By implementing the ‘KISS’ principle (‘keep it small and simple’), the author emphasized the design of a user friendly software that can be operated intuitively and which doesn’t require any advanced computer skills [Hohenwarter, 2006b, p. 81].

Virtual manipulatives: GeoGebra allows the creation of web-based interactive instructional materials, so called *dynamic worksheets*. Unlike other educational software, GeoGebra has no restrictions whatsoever concerning the export of dynamic figures into web-pages, and these can even be easily edited and modified later on if necessary. “Such customised interactive worksheets have led many teachers to foster experimental and discovery learning for their students and to share thousands of such worksheets on the GeoGebraWiki.” [Hohenwarter and Lavicza, 2007, p. 51]

Platform independence: Since GeoGebra is programmed in Java, it runs on virtually any operating system by just requiring a Java plug-in [Hohenwarter and Lavicza, 2007, p. 51]. GeoGebra can be used on MS Windows computers as well as MacOS computers without any problems. Additionally, all operating systems can run the same version of GeoGebra which prevents delays of software releases for different operating systems as often seen for commercial products.

International user community: During the last few years, “GeoGebra has been rapidly gaining popularity among teachers and researchers around the world” [Hohenwarter and Lavicza, 2007, p. 54]. Currently (Spring 2008), the GeoGebra web-page receives about 300,000 visitors from 194 different countries per month, and the software has been translated by volunteers to 39 different languages (currently) allowing students to use it in their local languages. Furthermore, the self-supporting international user community “shares free interactive teaching materials on the GeoGebraWiki and supports fellow users through the user forum.” [Hohenwarter and Lavicza, 2007, p. 51]

4.2 Review of Introductory Materials for DGS

In order to find out about how dynamic geometry software is commonly introduced to novices, different types of introductory materials for dynamic geometry software were reviewed. The programs *Cabri II Plus* and *Geometer’s Sketchpad* were selected from the wide range of dynamic geometry software because they are the most widely used programs in Europe (Cabri) and the USA (Geometer’s Sketchpad).

4.2.1 Summaries of Introductory Materials

The following summaries review the structure of these introductory materials as well as the nature of available help provided for introducing the software programs mentioned above to novices.

Cabri Geometry II Manual

The *User Manual for Cabri Geometry II* [Capponi, 2001] is a 50 pages document that is supposed to enable the user to gather first experiences with the software [Capponi, 2001, p. 2]. The manual contains a section called *First Steps in Cabri-Geometry II* which consists of eight topics meant to introduce the basic features of the software to novices.

After giving an overview of the layout of the user interface, the general use of the Cabri toolbar is explained. The different toolboxes are introduced and information about how to open a toolbox, activate a certain tool, and access its tool help is given. In addition, the mouse-over screen messages are explained.

The remaining six topics deal with the following mathematical contents which are designed to introduce a certain selection of different geometry tools as well as features of the software: triangle-based constructions, locus line, measurement transfer, transformations, analytical geometry, and graphical representation of functions. Additionally, users learn how to change certain properties of created objects and gain basic skills in how to operate the software in different mathematical situations.

Objects created: triangle, point, midpoint, line, segment, perpendicular line, perpendicular bisector, angle bisector, circle, locus, regular polygon, vector, number, function graph

Object properties: show label, add text, hide / show objects, measure distance and length, measurement transfer, show equation and coordinates

Other activities: open toolbox, activate tool, move objects, select objects, animation, translation, rotation, dilation, intersection of objects, create point on object

DGS features: display tool help, label objects ‘on the fly’, choose two out of three objects, change appearance of objects, show coordinate axes, define a coordinate grid, use the calculator, create a macro

All of the forgoing skills can be obtained by working through a variety of activities containing detailed instructions as well as additional challenging problems which should be figured out by the user. In order to support recreating the given examples, screen shots and sketches are included. All geometric tools and features are introduced by means of constructions and exercises, whereby (in most cases) several tools are used within each activity.

As expected from a manual, information about the menu items and a complete list of available tools is provided. The purpose of each tool as well as its use are explained. This enables users to search for missing information while trying out the software on their own.

Video Tutorials for Cabri Geometry II

The *Video Tutorials for Cabri Geometry II* [Cabrilog S.A.S, 2007] are part of the software’s help feature and can be accessed by selecting *Assistant* in the *Help* menu.

The first video called *Cabri Tour* provides an overview of the various features of the software. Several examples are presented and information about the software is given in audio format. In the section *Getting Started*, ten more tutorials are provided giving textual information in addition to the visual presentation, but no audio format is offered.

The tutorials deal with the introduction of geometric tools as well as features of the software: exploring and personalizing the toolbar, explanation of all available tools and creation of one’s own tools, triangle constructions and explorations, inserting images and graphing functions, transformations, changing preferences and export of dynamic figures. Again, a selection of geometric tools to create a variety of mathematical objects and certain

DGS features are introduced, while novices gain basic skills on how to use the software for different mathematical topics.

Objects created: triangle, perpendicular bisector, circle, segment, line, perpendicular line, point, midpoint, number, expression, regular polygon, polygon, vector

Object properties: activate tool, open toolbox, activate trace, hide / show objects, measure distance and length, show equations and coordinates

Other activities: move objects, intersection of objects, animation, insert images, apply expressions, change parameters, rotation, reflection, translation

DGS features: personalize toolbar, label objects ‘on the fly’, drag test, choose two out of three objects, create a macro, display tool help, use the calculator, show coordinate axes, change appearance of objects, export of dynamic figures, change preferences and settings

All of these skills can be learned by watching the videos which can be paused and resumed at any time, and by redoing the presented constructions and activities. Some tutorials contain detailed information and instructions about how to use the software, whereas others mainly give an overview of the software features.

Workshop Guide for Geometer’s Sketchpad

The *Geometer’s Sketchpad Workshop Guide* [Chanan et al., 2002] is a 55 page document designed to serve as a guide for professional development workshops. It can be used as a handout for workshop participants and contains multiple detailed hints and instructions on how to work with the software both for participants as well as for the workshop presenter.

The workshop guide consists of seven tours that deal with different mathematical topics and features of the software: constructing and exploring quadrilaterals, algebra applications, triangle centers, transformations, and trigonometric functions. Tools to create the following objects as well as certain DGS features are introduced. Again, workshop participants gain basic skills in operating the software and they are given the opportunity to explore a variety of mathematical situations.

Objects created: segment, circle, line, perpendicular line, parallel line, polygon, midpoint, angle, text, point, function graph, number, ray, locus, vector

Object properties: hide / show objects, show labels, measure length and angles, create captions, show coordinates, show trace

Other activities: select (all) objects, drag objects, intersect two objects, undo construction steps, animation, calculate, text formatting, reflection, translation, rotation, redefine (‘merge’) objects, create point on object

DGS features: save files, display coordinate axes, change preferences, create custom tools, script view of tools, multi-page document

Additionally, the workshop guide contains information about a series of sample files which demonstrate the potential of the software, as well as a collection of sample activities to introduce methods on how the software can be integrated into teaching and learning mathematics.

On the one hand, all skills mentioned above can be obtained by participating in a workshop where a presenter teaches the use of Sketchpad. On the other hand, novices could also work through the document independently. By reading through the various hints and explanations added for the instructor of a workshop, users also get access to professional advice and learn useful tricks about the software. A number of sketches and screenshots facilitate the reconstruction of examples. Additionally, each of the tours provides several challenge activities which allow users to practice newly developed skills that help to deepen the understanding of dynamic geometry.

4.2.2 Common Characteristics of Introductory Materials

Although the evaluated materials serve for the introduction of two different dynamic geometry software programs, several common characteristics emerged concerning the introduced tools to create certain mathematical objects, the presented mathematical contents, and DGS features.

User interface: The user interface of the corresponding software is explained in detail. Information about different components including the drawing pad, menubar, and toolbar is provided.

Geometric tools: Information about the organization of tools in so called toolboxes as well as instructions on how to open a toolbox and activate a tool are usually given before the actual activities start. Furthermore, users learn about different possibilities to obtain hints and help from the software itself.

Drag test: Dragging objects with the mouse, the key feature of dynamic geometry software, is usually introduced within the first activity and frequently used as the software introduction proceeds. Explanations about its influence on the mathematical content treated with dynamic geometry software and the new methods of investigating and discovering mathematical concepts are introduced.

Activities: A series of activities is presented to introduce a variety of dynamic geometry tools and features of the software. Usually several different tools are introduced in each activity. Additional features, such as how to change the appearance of objects, are often introduced ‘along the way’ within examples and tasks. At the end of each section, a selection of challenging examples is usually provided in order to allow users to practice their newly developed skills and find out more about similar applications of the software.

Content: The mathematical content presented in these materials covers a variety of difficulty levels. Although it varies with the introduction of different software programs, a common core pattern of treated elements includes:

- *Geometric constructions:* triangle and quadrilateral constructions as well as explorations of related mathematical concepts; transformations of objects
- *Analytic geometry:* creating objects within a coordinate system; displaying coordinates of points; dealing with vectors
- *Algebra:* defining parameters and exploring their influence on function graphs and their equations

4.3 Design of GeoGebra Introductory Workshops

In order to identify difficulties and impediments that occur during the introduction of a dynamic mathematics software like GeoGebra, the series of introductory workshops given during the MSP summer institute 2007 was evaluated. The overall design and content of the workshops were quite similar to the introductory materials for other dynamic geometry software programs reviewed in section 4.2. Additionally, several features that are unique in GeoGebra were also introduced. This allowed for inclusion of more activities that focused on concepts of algebra and calculus in addition to the ‘usual’ geometry tasks.

Although this series of introductory workshops was adapted to the needs of secondary school teachers in Florida, their content and design were based on experiences the presenter gathered by hosting a number of introductory workshops for GeoGebra in Europe and parts of the USA. Although this evaluation of introductory workshops could also have been realized in another place with participants of different nationalities, Florida was chosen because the MSP project mentioned before provided an ideal environment for the implementation of this study during a summer institute.

4.3.1 Objectives for Introductory Workshops

The four GeoGebra introductory workshops were designed in order to help the participating in-service secondary school teachers achieve the following objectives.

- Participants will become familiar with the basic use of GeoGebra (user interface, applying tools, changing properties of objects, ...).
- Participants will learn about where to get help and support for the use of GeoGebra.
- Participants will learn about common characteristics of paper-and-pencil constructions and dynamic geometry constructions (e.g. the *Line through two points* tool corresponds to a straight edge, the *Circle with center through point* tool corresponds to a compass).

- Participants will learn about fundamental differences between paper-and-pencil constructions and dynamic geometry constructions (e.g. a drawing is different from a construction).
- Participants will learn how to create basic geometric constructions (e.g. quadrilaterals, triangle centers).
- Participants will learn to apply transformations to objects (e.g. reflections, rotations).
- Participants will learn to insert pictures into a GeoGebra file.
- Participants will learn how to find out about the use of unfamiliar tools (e.g. toolbar help, online manual).
- Participants will learn to enter algebraic expressions (e.g. to create points, functions, conic sections).
- Participants will learn to use sliders to explore the impact of parameters on algebraic expressions and their graphical representations.
- Participants will learn how to use pre-defined commands in GeoGebra.
- Participants will learn to export dynamic figures as static pictures in GeoGebra.
- Participants will learn to create instructional materials by combining GeoGebra with a text processing software.

4.3.2 Structure and Content Overview

The following section gives an overview of the structure and content of the four introductory workshops evaluated in this study. Additionally, all tools and features introduced in each workshop are listed. Detailed information about the content of each workshop is given in the appendix (see chapter A).

In this series of introductory workshops, every session started with either an information input or the discussion of last days' home exercise. The main part consisted of four activities designed to help the participants to obtain the basic skills needed for independent use of GeoGebra. After each workshop the participants were supposed to work on a home exercise designed to practice their new skills and to apply their knowledge about GeoGebra outside the 'protected environment' formed by the workshops.

Workshop I: Basic Geometric Constructions

In workshop I the participants learned how to create basic geometric constructions in GeoGebra. They were introduced to a selection of dynamic geometry tools as well as to certain features of GeoGebra which facilitate the construction process.

Introduction: The presenter provided general information about the development, potential, and design of GeoGebra, and introduced the participants to additional user support available on the Internet.

Activity 1: Constructing a line bisector on paper using pencil, straightedge, and compass

Activity 2: Constructing a line bisector with GeoGebra

Tools introduced: Segment between two points, Circle with center through point, Intersect two objects, Line through two points, Move, Move drawing pad, Zoom in, Zoom out

Features introduced: Construction protocol, Navigation bar

Activity 3: Constructing a square over a given segment

Tools introduced: Polygon, Perpendicular line, Show / hide object

Activity 4: Constructing the circumscribed circle of an arbitrary triangle

Tool introduced: Line bisector

Feature introduced: Rename

Home exercise 1: Constructing an equilateral triangle

Workshop II: Angles, Transformations, and Inserting Images

In this introductory workshop the participants learned how to display angles in GeoGebra, as well as how to apply transformations to existing objects. Furthermore, they learned how to insert a picture into GeoGebra's graphics window and use it to enhance their dynamic figures.

Home Exercise Discussion: The home exercise of workshop I was discussed

Activity 1: Constructing a parallelogram and displaying its angles

Tools introduced: Parallel line, Angle

Features introduced: Grid, Point capturing, Context menu, Properties dialog

Activity 2: Creating a drawing tool for symmetric figures

Tools introduced: New point, Mirror object at line

Feature introduced: Trace on

Activity 3: Inserting a background image to explore axes of symmetry

Tool introduced: Insert image

Feature introduced: Background image

Activity 4: Rotating a polygon

Tool introduced: Rotate object around point by angle

Home exercise 2: Creating a drawing tool to check for axes of symmetry within a background image

Workshop III: Coordinates and Equations

In workshop III the algebra window and input field of GeoGebra were introduced. The participants learned how to enter coordinates and equations in order to create mathematical objects using their algebraic representations.

Home Exercise Discussion: The home exercise of workshop II was discussed

Activity 1: Entering, extracting, and modifying coordinates of points

Features introduced: Algebra window, free and dependant objects, coordinate axes, grid, labels of objects

Activity 2: Entering the slope intercept form of a linear equation

Tools introduced: Slider, Slope

Feature introduced: Redefine

Activity 3: Constructing a slope triangle

Tool introduced: Insert text

Command introduced: Slope

Features introduced: Static text, dynamic text, auxiliary objects

Activity 4: Creating a parabola and finding its vertex

Command introduced: Vertex

Home exercise 3: Exploring the parameters of a quadratic equation

Workshop IV: Functions and Export of Pictures

In workshop IV participants learned how to deal with functions in GeoGebra and how to explore calculus concepts. Furthermore, they learned how to export static pictures of their constructions and how to insert them into text processing software in order to create instructional materials for their students.

Home Exercise Discussion: The home exercise of workshop III was discussed

Activity 1: Entering polynomial functions

Commands introduced: Root, Extremum

Activity 2: Using GeoGebra's library of functions

Functions introduced: Trigonometric functions, absolute value function, logarithmic functions,...

Activity 3: Constructing the tangent to a function and displaying its slope function

Tool introduced: Tangent

Activity 4: Exporting pictures from GeoGebra and inserting them into an MS Word file

Features introduced: Export drawing pad to clipboard, Export drawing pad as picture

Home exercise 4: Creating a 'Function Domino' game

4.3.3 Implementation and Teaching Methods

In order to be able to actively participate in the introductory workshops, each participant was asked to bring along a notebook computer which allowed wireless Internet connection. On the first day of the summer institute, GeoGebra was installed on the participants' computers, and their owners were taught how to repeat this on their home computers in order to be able to work on the home exercises for each introductory workshop.

Additionally, the participants could access a web site that contained all workshop materials, as well as GeoGebra construction files for all workshop activities. Thus, they had the chance to practice at home as well as to review constructions that were created during the workshops in case they didn't save them on their notebook computers.

The activities of each workshop were presented in one of the following three ways. Since each group showed different group dynamics, the teaching methods varied throughout the day in order to match the groups' needs and technical skills.

Presentation: The instructor demonstrated the construction process and use of tools, commands, and features while participants were supposed to pay attention and take notes. Afterwards, they had time to redo the construction on their own or in cooperation with their colleagues. The instructor and helpers were available to answer questions and provided support if necessary.

Instruction: The instructor demonstrated a step-by-step construction while the participants were encouraged to work along with him. Thus, everybody tried to keep up with the pace of the presenter, although questions could be posed at any time. Additionally, helpers assisted if problems occurred or if very specific questions had to be answered.

Experiments: The instructor introduced a new task and encouraged the participants to find their own solution using GeoGebra. Again, the instructor and helpers were available to answer questions and help with technical problems. Afterwards, possible solutions were presented and discussed.

4.4 Evaluation Process for Introductory Workshops

4.4.1 Research Questions

The research study conducted in the context of this dissertation was implemented in order to find an answer to the following key research question:

Is it possible to identify common impediments that occur during the introduction process of dynamic mathematics software as well as to detect those especially challenging tools and features of the software GeoGebra in order to (a) provide a basis for the implementation of more effective ways of introducing dynamic mathematics software to secondary school mathematics teachers and (b) to design corresponding instructional materials for technology professional development?

The following series of more focused auxiliary questions was created in order to identify difficulties and impediments participants have to face when being introduced to the dynamic mathematics software GeoGebra during a technology workshop as well as to assess the usability of the software itself.

1. Are design, content, and difficulty level of the introductory workshops appropriate for secondary school teachers?
2. How do teachers experience the introduction to GeoGebra and what kind of feedback do they give concerning its usability?
3. Do users tend to subjectively rate GeoGebra's dynamic geometry tools to be of different difficulty levels when they are introduced in a workshop?
4. Do activities used to introduce dynamic geometry tools have impact on their subjective difficulty ratings?
5. (a) Is it possible to classify GeoGebra's dynamic geometry tools under groups of common characteristics that determine their general difficulty levels?
(b) Can the same classification criteria that determine the difficulty level of GeoGebra tools also be applied to the construction tools of other dynamic geometry software packages?
6. Do GeoGebra's features, algebraic input, or commands cause additional difficulties for the introduction of GeoGebra?
7. Do external variables such as math content knowledge, computer literacy, or the use of a touchpad, influence the subjective difficulty rating of GeoGebra, its tools, or its features?
8. Which difficulties, problems, and questions occur most often during an introductory workshop?

4.4.2 Evaluation Instruments

In order to find answers to the questions mentioned in section 4.4.1, a series of questionnaires was designed (see table 4.1), which were supposed to be filled in by every participant of the MSP summer institute. Since all questionnaires were labeled with an individual code¹, the data of each participant could be tracked without violating their anonymity.

Additionally, the helpers in each workshop group were asked to record all difficulties that occurred during the workshops by filling in so called ‘helper report cards’.

Day	Name	When / Where	Content
1	Survey I	Beginning of WS I	Computer literacy
1	Workshop I	End of WS I	Activities and tools
1	Home Exercise I	At home	Exercise and tools
2	Workshop II	End of WS II	Activities and tools
2	Home Exercise II	At home	Exercise and tools
3	Workshop III	End of WS III	Activities and tools
3	Home Exercise III	At home	Exercise and tools
4	Workshop IV	End of WS IV	Activities and tools
4	Home Exercise IV	At home	Exercise and tools
5	Survey II	Beginning of a WS	GeoGebra features
10	Survey III	End of Institute	Math content knowledge

Table 4.1: Survey distribution and content

Survey I: Computer Literacy

Survey I was designed to collect data about the participants and their everyday computer use in school and at home. It was supposed to generate a measure for the computer literacy of the participants and consisted of the following four sections (for complete survey see appendix, chapter B, pp. 230).

1. *General information*: gender, age, number of years as a teacher, grade levels and courses taught
2. *Technology use outside classroom*: computer use in days per week, activities for which the computer is used, software used in order to prepare lessons
3. *Technology use in classroom*: computer use in days per week, active computer use by students in days per week, how a computer is used in class, software / technology used in class
4. *Computer skills*: variety of activities to determine the level of computer literacy

¹Code: mother’s initials (maiden name), father’s initials, and four digits of the participants year of birth

Workshop Ratings

After each workshop, the participants filled in a workshop rating where they could rate the difficulty of the workshop activities and tools used during the workshop on a scale from 0 ('very easy') to 5 ('very difficult'). Additionally, they had the chance to give written feedback about the workshop (for complete forms see appendix, chapter B, pp. 233).

Home Exercise Ratings

For each home exercise, the participants received a form that allowed them to rate the difficulty of the home exercise on a scale from 0 ('very easy') to 5 ('very difficult'). Additionally, they were supposed to rate the GeoGebra tools used for the home exercise using the same scale and to record how much time they spent working on the exercise. Again, they had the chance to give written feedback on the home exercise (for complete forms see appendix, chapter B, pp. 237).

Survey II: GeoGebra Features

Survey II was supposed to collect additional data about the participants' computers used during the workshops. Participants also had the chance to rate the difficulty of the introduced GeoGebra features on a scale from 0 ('very easy') to 5 ('very difficult'), and to give written feedback about the complete series of GeoGebra introductory workshops (for complete survey see appendix, chapter B, p. 232).

Survey III: Mathematics Content Knowledge

Survey III was designed by a mathematics professor associated with the institute as a multiple choice test intended to measure the mathematics content knowledge of the summer institute 2007 participants. The survey consisted of 24 mathematical problems based on the Florida Sunshine State Standards² for middle school mathematics grades 6 - 8. Since the survey might be reused in the future, the original form can't be provided. Instead, three example questions are listed which are similar to the original problems.

- *Example problem 1:* Change the base ten number 42_{10} into base 3.
 (a) 1120_3 (b) 2122_3 (c) 0211_3 (d) 1021_3
- *Example problem 2:* Compute the Greatest Common Factor (GCF) of 990 and 6300.
 (a) 66 (b) 90 (c) 165 (d) 30
- *Example problem 3:* Simplify $\left(\frac{27n^9}{m^6}\right)^{-\frac{2}{3}}$
 (a) $-\frac{n^6}{9m^4}$ (b) $\frac{9n^6}{m^4}$ (c) $\frac{m^6}{9n^4}$ (d) $\frac{m^4}{9n^6}$

²Florida Sunshine State Standards: www.fldoe.org/bii/curriculum/ss/pdf/math6.pdf

Helper Report Cards

During every workshop, helpers filled in so called ‘helper report cards’ in order to record occurring difficulties and questions. They had to classify the problem (GeoGebra, MS Windows or MacOS computer, mathematics content) and briefly describe its nature.

4.4.3 Statistical Tests Used

For the analysis of the data collected, spreadsheets (MS Excel) as well as the predictive analytics software SPSS were used in order to carry out statistical tests. Since the number of participants was not sufficiently large to guarantee normally distributed data, parametric tests like the t-test, ANOVA, and Pearson’s correlation coefficient which require continuous and normally distributed data couldn’t be used [Field, 2006, p. 287 and p. 324]. Since the distribution of data is partly nominal and partly ordinal with discrete values, non-parametric tests were used as equivalents to the parametric tests mentioned above: Mann-Whitney test, Wilcoxon signed-rank test, Kruskal-Wallis test, and Spearman’s correlation coefficient [Field, 2006, p. 522 – 556]. In order to analyze answers to the open-ended questions as well as the helpers reports, a Grounded Theory approach [Cohen et al., 2005, p. 150 – 152] was implemented which allowed systematic organization and review of the written feedback by assigning keywords and organizing them in different categories.

Chapter 5

Description of Workshops and Participants

This chapter summarizes the demographic data collected from the workshop participants and describes their computer using habits and skills. Furthermore, the difficulty ratings of workshop activities and home exercises were analyzed, and the open feedback from participants as well as the helper reports were evaluated for every workshop.

5.1 Workshop Participants

5.1.1 Demographic Data of Participants

A total number of 44 secondary school mathematics teachers participated in the evaluation of GeoGebra introductory workshops during the MSP Summer Institute 2007 (see section 4.1). The group was composed of 35 women and 9 men with an average age of 38.62 years. The youngest teacher participating was 24 years old, while the oldest participant aged 59.

Although the teaching experience ranged from 1 year to 29 years, 54.8% of the participants taught for five years or less in secondary schools. In 14.3% of the cases, the teachers worked 15 or more years in their profession. Concerning the grade levels taught by each teacher, the following frequencies were recorded: 25 teachers taught one grade level (61.0%), 8 participants taught students from two different grade levels (19.5%), and 7 teachers worked with 3 different grade levels (17.1%). One of the teachers even taught 4 different grade level courses. 18 participants taught pre-algebra, 20 taught algebra courses, 2 teachers taught pre-calculus as well as calculus, and 9 people taught geometry classes.

30 participants (73.2%) taught in middle schools (grade levels 5 through 8), whereas 63.3% exclusively worked with students of one grade level. 11 participants (26.8%) were high school teachers (grade levels 9 through 12), whereof 54.5% also taught just one grade level (see table 5.1).

School	Percentage	1 grade	2 grades	≥ 3 grades
Middle school	73.2%	63.3%	13.3%	23.3%
High school	26.8%	54.5%	36.4%	9.1%

Table 5.1: Number of grade levels taught by middle and high school teachers

5.1.2 General Computer Use

At Home and in School

On average, the participating teachers use a computer at home on 5.87 days of the week, whereas 64% of the participants stated they use it every day (including the weekend). Additionally, 94.3% of the teachers use a computer on every week day in school (e.g. preparation of lessons, record keeping). Although 14.6% of the participants never use a computer for teaching, 41.5% use it once or twice, while 14.6% of the teachers even use a computer three or four times per week during their classes. 29.3% of the participants stated they use a computer for teaching every day, while 59.5% of all teachers allow their students to actively use a computer in class at least once a week.

Table 5.2 lists the percentages of teachers using the computer for a selection of activities at home and at school (outside the classroom). It shows, that most of the teachers use a computer for communication, preparation of lessons and teaching materials, as well as to keep records of their students both at home and at school.

Activity performed	At home	In school
Check e-mail	95.3%	100%
Chat with friends	55.8%	25.6%
Prepare lessons	81.4%	95.2%
Create own teaching materials	85.7%	85.4%
Look for teaching materials on the Internet	90.7%	90.5%
Look for content information on the Internet	95.2%	88.1%
Keep records of students' grades	58.5%	97.6%

Table 5.2: Teachers' computer use at home and at school

Concerning the use of computers for teaching, 73.2% of the teachers stated they use the computer in combination with a projector as a presentation tool. 64.3% of the participants allow their students to practice for the *FCAT*¹, while 34.1% of the teachers let their students play educational computer games as a reward. 70.7% of the teachers reported they let their students use computers in order to discover mathematical concepts.

¹Florida Comprehensive Assessment Test

Software Used

The most widely used type of software for the preparation of lessons is word processing software (92.7%), followed by presentation software (65.9%), and spreadsheets (48.8%). Only 14.6% of the teachers also use dynamic geometry software and 12.2% of the participants use computer algebra systems in order to prepare their lessons (see table 5.3).

Concerning teaching, 71.4% of the participants work with presentation software and 78.6% with word processing software in their classrooms. Spreadsheets are used by 43.9% of the teachers, whereas only 19.0% use dynamic geometry software and 4.9% computer algebra systems for teaching (see table 5.3).

Software used	Lesson preparation	Teaching
Presentation software	65.9%	71.4%
Word processing software	92.7%	78.6%
Spreadsheets	48.8%	43.9%
Dynamic geometry software	14.6%	19.0%
Computer algebra systems	12.2%	4.9%

Table 5.3: Software used for lesson preparation and teaching

Considering the relatively low percentages for the use of dynamic geometry software and computer algebra systems, the need for professional development to facilitate the use of mathematical software is obvious. Although teachers use computers frequently for a wide range of activities, most of them seem to be unaware of the potential of educational software for teaching and learning mathematics.

Hardware Used during Introductory Workshops

During the four introductory workshops, 43.9% of the participants used their own notebook computers while most of the other teachers used notebook computers provided by their schools. These borrowed computers often showed restrictions concerning the user rights (e.g. saving files, installing software) and therefore, caused additional problems.

56.1% of the participants used a MacOS notebook computer, while the others used MS Windows notebook computers. All five participants who had major problems with their computers in general, used MacOS notebook computers: one participant owned the computer and four teachers used computers owned by their schools.

Most of the participants (82.1%) used a touchpad instead of a computer mouse in order to operate GeoGebra. Hardware differences between MacOS and MS Windows notebook computers need to be considered. While mouse users can decide if they would like to use a mouse with one or two keys, touchpad users depend on the number of keys built in with the touchpad of their notebook computers. 17 out of 23 teachers with MacOS notebooks used touchpads, meaning that just one key was available for them. 15 out of 18 participants

with an MS Windows notebook computer used their touchpads too, which provided 2 keys for them.

5.2 Workshop Evaluation

Since the participating teachers rated the difficulty of the workshop activities on a scale from 0 (‘very easy’) to 5 (‘very difficult’) every day, the average difficulty level of each workshop could be determined. Although the content of the workshop series was designed to be increasingly demanding, all workshops were rated to be of a similar difficulty level with average ratings between $\bar{x} = 1.46$ for workshop I and $\bar{x} = 1.81$ for workshop II (see table 5.4).

Considering the rating scale from 0 to 5, this implies that on average the participants thought the workshops to be rather ‘easy’. Therefore, the workshop contents seem to be of appropriate difficulty level for the target audience composed of secondary school mathematics teachers.

	Workshop	Rating
I	Basic Geometric Constructions	1.46
II	Angles, Transformations, and Pictures	1.81
III	Coordinates and Equations	1.60
IV	Functions and Export of Pictures	1.70

Table 5.4: Average ratings of introductory workshops

5.2.1 Workshop Activities Ratings

Table 5.5 lists the activities offered in the four workshops as well as their average difficulty ratings, whereby none of the activities were rated more difficult than $\bar{x} = 2.05$ (workshop II, activity 4 ‘Rotation of a polygon’). In order to find possible explanations for the difficulty ratings of particular workshop activities, experiences with the participants obtained during the workshops will be considered in addition to the results of the statistical data analysis of the workshop ratings.

Workshop I

In workshop I, activity 2 ‘Line bisector with GeoGebra’ was rated the ‘easiest’ GeoGebra task of all workshops with an average rating of $\bar{x} = 1.26$. Since the construction of a line bisector was practiced on paper before the teachers even started GeoGebra, the content of the activity was well known and therefore caused no major difficulties. Although most of the participants used the software for the first time ever, they seemed to get along with it pretty well.

Workshop I		Rating
1	Line bisector on paper	1.02
2	Line bisector with GeoGebra	1.26
3	Square	1.93
4	Circumscribed circle of a triangle	1.63
Workshop II		
1	Parallelogram with angles	1.60
2	Drawing tool for symmetric figures	1.84
3	Inserting a background image	1.74
4	Rotation of a polygon	2.05
Workshop III		
1	Coordinates of points	1.30
2	Linear equation	1.65
3	Slope triangle	1.78
4	Parabola	1.68
Workshop IV		
1	Polynomial functions	1.95
2	Library of functions	2.00
3	Tangent and slope function	1.63
4	Export of static pictures	1.46

Table 5.5: Average ratings of workshop activities

Activity 3 ‘Square’ was the most challenging task of this workshop with an average rating of $\bar{x} = 1.93$. Only 40.5% of the participants rated this activity ‘easy’ (rating 0 or 1) while 21.4% of the teachers thought it to be rather ‘difficult’ (rating 4 or 5). A possible reason for this is that many participants didn’t know how to construct a square in the first place. Therefore, the use of GeoGebra was not only new for them, but also the content of the task, which caused additional difficulties for the teachers. Furthermore, the differences between drawings and ‘real’ constructions were explained at this point. This seemed to be a difficult concept to grasp for some of the participants, but this distinction is essential for an effective use of a dynamic geometry system for teaching and learning mathematics.

Workshop II

Activity 1 ‘Parallelogram with angles’ was rated the easiest task of workshop 2. Again, this task was a quadrilateral construction, but this time the participants seemed to be prepared to think about the properties of the parallelogram in order to select appropriate tools and order of construction steps. They also seemed to have understood the concept of construction vs. drawing and started to use the drag test in order to check their quadrilateral constructions.

The last activity of this workshop ‘Rotation of a polygon’ was rated to be the most difficult task of all workshops with an average rating of $\bar{x} = 2.05$. The construction consisted of five different types of objects (polygon, point, circle, segment, angle) and required the use of seven different tools, more than the number used in earlier activities. Furthermore, in all three workshop groups the amount of time left for this last activity was less than initially scheduled. Therefore, the presenter had to rush in order to finish the construction in the allotted time, which caused many teachers to just watch the construction process instead of working along with the group.

Workshop III

In workshop III the first task ‘Coordinates of points’ was rated to be the easiest task with an average rating of $\bar{x} = 1.30$. This activity was meant to introduce algebraic input and therefore, didn’t deal with challenging mathematical content in order to facilitate the first use of the algebra window and input field of GeoGebra.

By contrast, activity 3 ‘Slope triangle’ was rated to be the most difficult task of this workshop with an average rating of $\bar{x} = 1.78$. In this activity, a geometric construction was combined with algebraic input for the first time ever, whereby several computations were needed in order to create dynamic text within the graphics window. Thus, the activity was rather complex and challenging for the participants.

Workshop IV

The very last activity of this workshop series ‘Export of static pictures’ was rated to be the easiest task of workshop IV with an average rating of $\bar{x} = 1.46$. It didn’t require the creation of a new dynamic figure in GeoGebra, but dealt with the export possibilities of static pictures in GeoGebra and the insertion of pictures into an MS Word file.

By contrast, activities 1 ‘Polynomial functions’ and 2 ‘Library of functions’ seemed to be the most challenging activities for the participants with average ratings of $\bar{x} = 1.95$ for the first and $\bar{x} = 2.00$ for the second activity. Both tasks involved the mathematical concept of functions and their characteristics, which probably exceeded the average middle school mathematics content familiar to the majority of the workshop participants.

5.2.2 Workshop Feedback

On the workshop rating sheets the participants were asked to explain what they liked about the corresponding introductory workshop and if there was anything they didn’t like. On survey II the participants were asked to give general feedback about the series of workshops (for exact wording of the questions see appendix, chapter B, pp. 232).

The purpose of these questions was to find out about the general attitude of participants towards the introductory workshops in general, and the use of GeoGebra in particular. Additionally, more specific information about difficulties concerning workshop activities as well as about the use of GeoGebra tools and features was supposed to be gathered.

General Review of Responses

Table 5.6 lists the number of participants as well as the number of responses per workshop rating for both open ended questions. The ratio between positive answers (“*What did you like...?*”) and negative answers (“*Is there anything you didn’t like...?*”) is about 7 to 1 in the first workshop, compared to about 4 to 1 in the other three workshops. Based on the number of positive responses the conclusion was drawn, that the overall attitude towards the introductory workshops as well as GeoGebra was definitely positive.

By comparing the number of words per response a rather high number of 14 words per negative response was recorded for workshop II, a consequence of two participants explaining their problem in exceptional detail (32 words per answer). In general, the responses were kept rather short with about 9 words per positive response and an average of 8 words per negative response.

	Workshop I		Workshop II	
	like	didn’t like	like	didn’t like
Total participants	42	42	43	43
Total responses	40	6	40	11
Responds in %	95%	14%	93%	26%
Total words	395	36	340	156
Words / response	10	6	9	14
	Workshop III		Workshop IV	
	like	didn’t like	like	didn’t like
Total participants	41	41	41	41
Total responses	32	9	36	8
Responds in %	78%	22%	88%	20%
Total words	286	66	278	38
Words / response	9	7	8	5

Table 5.6: Workshop feedback

Table 5.7 displays the same data for the feedback collected by survey II. Although just two thirds of the participants gave written feedback, the average number or words per response was with an average of 26 much higher than the average number of words for open responses to the workshop ratings. Here, the participants tended to give more detailed feedback about the workshops and GeoGebra, as well as summaries of their experiences with the software. Additionally, some of them wrote about their intention to use GeoGebra in their classrooms in order to enhance their teaching of certain topics, as well as to help their students understand unusually challenging mathematical concepts better.

	Survey II feedback
Total participants	41
Total responses	27
Responds in %	66%
Total words	707
Words / response	26

Table 5.7: Workshop series feedback

Detailed Review of Responses

In order to gain an overview of the general feedback given to the open ended questions mentioned above, the responses of participants were systematically analyzed based on a Grounded Theory approach [Strauss and Corbin, 1998].

After splitting the responses into the two different topics *Introductory Workshops* and *GeoGebra*, keywords were assigned to each response according to its content. Then, the 11 keywords related to the introductory workshops were grouped in four categories ‘Workshop Design’, ‘Workshop Contents’, ‘Workshop Activities’, and ‘Workshop Implementation’. The 15 keywords concerning GeoGebra were grouped in four categories ‘Characteristics of GeoGebra’, ‘GeoGebra Tools and Features’, ‘Algebraic Input in GeoGebra’, and ‘Teaching with GeoGebra’. Thus, the frequencies of occurrence could be calculated for each keyword giving an overview of the participants’ attitude towards the introductory workshops in general and GeoGebra in particular (for explanations of the keywords see appendix, sections C.1 and C.2).

Introductory Workshops Feedback

In this section, the participants’ open feedback is summarized in order to provide a general idea about their attitude towards the introductory workshops. Table 5.8 shows the coded feedback of the workshop participants, arranged in categories and keywords, as well as the frequencies of occurrence for each keyword related to the series of *Introductory Workshops*.

Workshop Design: In this category feedback about workshop components is summarized that seemed relevant to the participants or were addressed with improvement suggestions.

- *Activities:* Four responses stated that the workshop participants liked the ‘*hands on*’ activities as well as the discussions of home exercises. Thus, these two components seem to be important for introductory workshops and therefore should be strengthened by allowing more time for active participation and discussion in each workshop.
- *Support:* Eight responses reported that teachers liked to having ‘*plenty of helpers*’ around who answered questions and offered support during the work-

Introductory Workshops Feedback				
<i>Category</i>		<i>Keyword</i>		<i>Frequency</i>
I a	Workshop Design	1	Activities	4
		2	Support	8
		3	Documentation	11
I b	Workshop Contents	4	Contents	6
		5	Examples	7
I c	Workshop Activities	6	Constructions	21
		7	Algebra	12
I d	Workshop Implementation	8	Presentation	13
		9	Pace	16
		10	Practice time	30
		11	Popularity	26

Table 5.8: Coding and frequencies for feedback about *Introductory Workshops*

shops. Thus, most of the participants were able to ‘*follow along*’ with the presenter and the number of interruptions of the presentation due to urgent questions was minimized.

Workshop experience indicates that at least one assistant should be available during an introductory workshop in addition to the presenter. Furthermore, participants should be asked to address their neighbors first when questions arise, before they ask the assistant for help or interrupt the presentation by asking the instructor. Additionally, the instructor should frequently ask about problems or questions throughout the workshop pausing the presentation in order to allow discussion among the participants, as well as to offer support with difficulties.

- *Documentation*: Eleven responses pertained to missing documentation for the workshops, referring to workshop ‘*handouts with detailed instructions*’ for every activity. Participants wanted to use them during the workshops in case they got lost, as well as to ‘*review the workshop contents at home*’ and get assistance with the home exercises.

These requests should definitely be considered when reviewing the workshop materials used for this series of introductory workshops, because such documentation will reduce the level of confusion and frustration for the participants and will assist them when working with GeoGebra by themselves.

Workshop Contents: In this category feedback about the workshop contents, activities, and examples used during the presentation is summarized.

- *Content*: Four responses stated that the teachers enjoyed learning how to use GeoGebra. They were ‘*excited to learn*’ many new things and requested specific

content workshops for middle and high school level mathematics topics. Two teachers complained that there was *'too much information'* presented during the workshops and that it was *'hard'* for them to *'memorize everything later on'*.

Concerning the amount of content information, proper documentation in form of handouts with detailed instructions could make it easier for the teachers to keep an overview about what they learned during the workshops. Especially for American teachers, special content workshops related to certain mathematics courses they teach should be developed in order to meet the teachers' needs for everyday teaching and to help them transfer workshop contents into their teaching practice.

In order to help presenters adjusting workshop contents to the needs of a particular audience, workshop materials and documentation could be designed as introduction and application topics. While the introduction topics contain basic information about the use of GeoGebra combined with a variety of relatively easy mathematical content, the application topics are designed for special mathematical content and different levels both of mathematical skills as well as use of GeoGebra. Based on the prerequisites for each topic and the technical and mathematical skills of the participants, presenters can finally select a series of topics in order to adapt the workshop contents to the participants' needs.

- *Examples*: Seven responses were related to the examples shown by the presenter to increase the participants' motivation as well as to show GeoGebra's potential and versatility. Teachers stated that they *'liked the examples'* and some suggested that even more of them could be shown. Additionally, participants also wanted to actively create examples which were similar to the workshop activities in order to *'get more practice'* using GeoGebra.

Although the time frame of introductory workshops usually doesn't allow for creation of a series of similar examples dealing with the same content, a practice block for each workshop topic providing a choice of tasks for the participants could improve this situation. Working on selected tasks either alone or with a colleague, participants could practice their new skills by trying out similar tasks, or they could enhance their knowledge by dealing with more challenging exercises.

Workshop Activities: In this category feedback about the geometry and algebra workshop activities is summarized.

- *Constructions*: In 21 responses teachers stated that they liked creating different geometric constructions with GeoGebra. Although the construction process for some of the figures was unfamiliar to the teachers, they *'enjoyed'* using technology and learning how to integrate the properties of a geometric object into its construction process. Participants especially liked the transformation activities of workshop II because they learned to *'create appealing exercises'* for their students.

Although creating geometric constructions isn't an important topic in the American curriculum (compared to Austria, where geometric constructions are covered in detail during the middle grades), teachers obviously enjoyed the activities involving geometric constructions. They could actively use knowledge about geometric properties in order to construct the corresponding quadrilaterals and triangles while they were introduced to the basic tools of GeoGebra. Nevertheless, participants complained that there were too many construction steps necessary to get the final product. This was probably caused by their unfamiliarity with the content and the construction process itself. Again, this could be addressed by providing detailed instruction handouts during the GeoGebra workshops.

- *Algebra*: 12 responses contained feedback about algebra related activities, especially the ones dealing with graphing lines and exploring their slope, as well as finding the graphical solution of a system of equations. Some teachers stated, that these concepts were '*much easier to visualize*' with GeoGebra, and therefore, easier to teach and understand for their students.

When working on the algebra activities, teachers realized right away GeoGebra's potential to support and facilitate their teaching of algebraic concepts. They saw a way to transfer their new abilities to their classrooms and help their students to understand unusually challenging concepts by using dynamic visualizations.

Workshop Implementation: In this category feedback about the implementation of the introductory workshops (e.g. presentation style and time management) is summarized.

- *Presentation*: 13 responses pertained to the presentation style of the workshop instructor. Teachers stated that it was '*easy to follow along*' because the instructor '*took time to thoroughly explain*' and '*model construction processes*'. They liked the visualizations and step-by-step instructions, as well as watching the construction process first and then redoing it afterwards.
- *Pace*: 16 responses contained feedback about the pace of the presentations. Twelve responses were positive, stating that it was '*easy to keep up*'. The remaining four teachers complained that the presentation pace was '*too fast*' for them, and that they would have needed more time to be able to keep up with the instructor.

Although the pace didn't fit everybody's needs, the majority of participants seemed to be content with the workshop presentations. Since it is very difficult, if not impossible, to design a workshop that works perfectly for every single participant with individually different computer skills, content knowledge, and personal situation, compromises need to be made. On the one hand, workshops need to contain blocks led by the instructor wherein participants try to follow along, learn new skills, and get information. On the other hand, practice blocks

should be included allowing participants to practice their new skills and process their new knowledge by selecting tasks of their individual interest from a pool of provided practice activities. The participants should get the chance to work in pairs of similar pace and interest, as well as to get support from the instructor and assistant whenever necessary. Again, detailed instructional materials should be provided in order to support the teachers and facilitate their first use of GeoGebra.

- *Practice time*: 30 responses contained feedback about the time to practice new skills during the workshops. 21 teachers asked for ‘*more time*’ to spend on the activities or just ‘*playing with GeoGebra*’ in order to explore its functionality and potential. Participants also stated that they needed more time to ‘*effectively grasp introduced concepts*’. In the remaining nine responses, teachers reported that the workshops were ‘*too short*’ for them and that they wished to spend more time getting to know additional features of GeoGebra.

In this case, two groups of participants can be identified. While one group of teachers would like to have more time for the same amount of content, the other group of participants would like to have additional workshop time in order to advance their knowledge about GeoGebra and its use in classrooms. After introducing the basic use of GeoGebra, it therefore would make sense to offer different follow-up workshops, where either already introduced concepts can be deepened and practiced, or additional and advanced contents can be discussed.

- *Popularity*: 26 responses were related to the popularity of the introductory workshops. Teachers stated that they enjoyed the workshops, found them ‘*awesome*’ and ‘*very interesting*’.

Thus, the conclusion was drawn that the majority of participants were content with the content and implementation of the GeoGebra introductory workshops.

GeoGebra Feedback

Table 5.9 displays the coded feedback of the workshop participants, arranged in categories and by keywords, as well as the frequencies of occurrence for each keyword related to the use of *GeoGebra*. Below, the feedback is summarized in order to provide an overview about the participants’ attitude towards GeoGebra itself.

Characteristics of GeoGebra: In this category, characteristics attributed to GeoGebra by the participating teachers, as well as general feedback about the software are summarized.

- *User friendly*: In 34 responses teachers described GeoGebra as ‘*user friendly*’ and ‘*teacher friendly*’, or positively mentioned its ‘*usability*’. They called the software ‘*easy to use*’, ‘*easy to learn*’, or ‘*simple to understand*’, and stated that GeoGebra ‘*makes drawing easier*’.

GeoGebra Feedback				
<i>Category</i>		<i>Keyword</i>	<i>Frequency</i>	
II a	Characteristics of GeoGebra	1	User friendly	34
		2	Useful	10
		3	Potential	5
		4	Dynamic	14
		5	Feedback	28
II b	GeoGebra Tools and Features	6	Use of tools	13
		7	Helpful features	6
		8	Images & text	14
		9	Export	11
II c	Algebraic Input in GeoGebra	10	Algebraic input	10
		11	Functions	8
II d	Teaching with GeoGebra	12	Classroom use	33
		13	Methods	15
		14	Applications	13
		15	Materials	8

Table 5.9: Coding and frequencies for feedback about *GeoGebra*

Considering this variety of positive answers concerning the user friendliness of GeoGebra the developer seems to have reached the goal of creating a software that is easy and intuitive to use for teachers and students.

- *Useful*: 10 responses referred to the usefulness of GeoGebra for teaching mathematics. Teachers stated that the software could potentially be ‘*helpful*’ for their teaching and appreciated its free availability for teachers and students in school and at home.

Noticing that GeoGebra could be useful for their everyday teaching definitely increases the likelihood of teachers actively using the software in their classrooms. Although not every single teacher who participated in the workshop series is expected to actually integrate the dynamic mathematics software into their everyday teaching, the majority of participants at least seemed to consider this option.

- *Potential*: In 5 responses teachers mentioned the potential of GeoGebra for teaching a variety of mathematical concepts and different levels, as well its versatility and flexibility. They called it a ‘*powerful tool*’ for mathematics teaching and learning.
- *Dynamic*: 14 responses pertained to the fact that GeoGebra is ‘*dynamic*’ and ‘*interactive*’. Teachers enjoyed creating dynamic figures that allowed them to visualize changes by moving objects with the mouse, as well as the ability to demonstrate the impact of a single component on the whole figure.

During the introductory workshops, teachers got the chance to discover the advantages of a dynamic and interactive learning environment for their teaching and their students' learning. Since less than 20% of participants were familiar with dynamic geometry software (see section 5.1.2) before they attended the workshop series about GeoGebra, this concept was new for most of the teachers and seemed to really impress them.

- *Feedback*: 28 responses were related to general feedback about GeoGebra. Teachers stated that they '*enjoyed*' using the software and that '*GeoGebra made the time fly*'. They called GeoGebra a '*remarkable*' and '*exceptional*' tool for mathematics teaching and learning.

Again, those statements support the impression, that teachers accept GeoGebra very well as a new teaching tool. They seem to notice its potential value for their everyday teaching and appreciate its user friendliness as well as its flexibility and diversity of application possibilities.

GeoGebra Tools and Features: In this category, feedback about the tools and features of GeoGebra, including inserting images and text, as well as export possibilities of static pictures, is summarized.

- *Use of tools*: 13 responses were related to the use of GeoGebra tools. Teachers liked the *Angle* tool and its application to find the '*interior angles of polygons*', as well as the *Slider* tool especially to '*change parameters of linear equations*'.
- *Helpful features*: In 6 responses teachers gave feedback about different GeoGebra features. They especially liked the *Toolbar help*, because it was useful to find out how a certain tool could be applied, the *Properties dialog*, which allowed them to customize the appearance of their dynamic figures, as well as tracing objects. Also, the *Undo* button was very helpful for them when trying to create constructions and having to '*undo mistakes*'.
- *Images and text*: 14 responses were related to inserting images and static, as well as dynamic text into the graphics window of GeoGebra. Teachers appreciated that they were able to create appealing activities for their students by using, for example, images for transformations, or inserting dynamic equations in order to show how changing the slope of a line affects its equation.
- *Export*: In 11 responses teachers stated that they liked GeoGebra's export possibilities for static pictures. They liked the fact that they could put them '*on tests and notes*' as well as to create worksheets for their students.

Overall, the feedback for GeoGebra's tools and features was very positive. Teachers found them easy to use and appreciated their functionality. Features that allow them to enhance their dynamic figure (e.g. *Properties dialog*, inserting images and text) were especially popular among the participating secondary school teachers.

Algebraic Input in GeoGebra: In this category feedback about the capability of GeoGebra to deal with algebraic expressions is summarized, which is the main difference when compared to pure dynamic geometry software and opens up an additional field of applications for GeoGebra.

- *Algebraic input:* 10 responses were related to GeoGebra's potential for teaching algebra in secondary school. Teachers stated that they liked to enter algebraic expressions into the input field. They also noticed that GeoGebra could '*help deliver concepts of algebra*' and '*enhance students' learning*' of these unusually challenging topics.

In this case, teachers were mainly referring to graphing lines, exploring the slope of lines, as well as visualizing the solution of equations in one variable by splitting them up into two functions. These concepts are usually taught in American middle schools, whereby a lot of students find them really challenging. Often, they lack the visual representation of these concepts and treat them in a purely abstract mathematical way. Thus, teachers liked the dynamic figures they created and saw their potential to facilitate their teaching, as well as for supporting their students' learning and understanding of algebraic concepts.

- *Functions:* In 8 responses teachers reported that they liked the function graphing capabilities of GeoGebra, especially in terms of polynomial functions.

Teaching with GeoGebra: In this category feedback the teachers gave about their ideas of actually using GeoGebra in their classrooms is summarized.

- *Classroom use:* 33 responses were related to the classroom use of GeoGebra. Teachers stated that they wanted to use GeoGebra for teaching mathematics and that they thought about integrating the software into their everyday teaching. They planned to use the software for visualization purposes as well as lesson enrichment, and they determined that it would help them to grasp their students' attention.

Although only two teachers actually thought about using GeoGebra after the first workshop, this number increased on the second day when already 11 teachers mentioned their plans about integrating GeoGebra into their teaching practice. This also indicates that teachers with little GeoGebra experience imagine themselves using the software and expect it to facilitate their everyday teaching. They want their students to benefit from the visualization potential of GeoGebra, and they hope that the software would facilitate dealing with concepts that are unusually hard to grasp for their students.

- *Methods:* In 15 responses teachers wrote about teaching methods that could be implemented in their classrooms when integrating GeoGebra. 5 teachers wanted to use it as a demonstration tool and stated that it would be '*easy to use as a teaching tool*'. The remaining 10 responses were related to the potential of GeoGebra to visualize mathematical concepts.

These statements indicate that teachers with little GeoGebra experience primarily see the software as a presentation tool that can facilitate their teaching by visualizing mathematical concepts in a dynamic way. None of the teachers considered letting their students actively use GeoGebra in order to discover mathematics on their own. Nevertheless, using GeoGebra as a presentation tool is the first step towards a successful integrating of technology into everyday teaching. Once the teachers get comfortable with using the software for visualization and presentation purposes, chances are that they will allow their students to actively use either the software itself or prepared materials that foster discovery learning as well as mathematical experiments.

- *Applications*: 13 responses dealt with the potential of GeoGebra to be used for a wide range of mathematical topics. Teachers appreciated the software's '*powerful application possibilities*' as well as its '*flexibility*' to visualize mathematical content.

During workshops II and III, nine teachers noticed that GeoGebra supports a variety of mathematical concepts that range from geometry to algebra and calculus. They liked the versatility of the software which exceeds the potential of pure dynamic geometry software and therefore allows them to use GeoGebra for secondary school mathematics throughout all grade levels.

- *Materials*: In 8 responses teachers reported that they liked to create their own teaching materials with GeoGebra by exporting pictures from GeoGebra and inserting them into an MS Word file.

In workshop IV teachers were introduced to another application possibility of GeoGebra, namely the creation of static instructional materials. Since American teachers in general are used to having prepared materials to work with (compared to Austria, where many teachers create their own teaching materials) this feature of GeoGebra allows them to develop more independence from the published textbooks and to create and use materials that better match their students needs in certain situations.

5.3 Home Exercise Evaluation

5.3.1 Home Exercises vs. Workshop Activities

In addition to the rating of workshop activities, participants also rated the difficulty level of home exercises of each day on a scale from 0 ('very easy') to 5 ('very difficult'). Table 5.10 lists the average difficulty ratings of the four home exercises and workshops of the same day, as well as their standard deviations.

Compared to the average difficulty ratings of the corresponding workshop activities, three out of four home exercises were rated more difficult indicating that either the difficulty and nature of the home exercises weren't appropriate for the knowledge and skills of

Home exercise		Mean	StDev	WS	Mean	StDev
1	Equilateral Triangle	1.42	1.36	I	1.46	1.40
2	Drawing Tool ...	2.43	1.39	II	1.81	1.31
3	Quadratic Equation	2.10	1.50	III	1.60	1.22
4	'Function Domino' Game	2.80	1.16	IV	1.70	1.31

Table 5.10: Average ratings of workshops and home exercises

the participants, or that the use of GeoGebra outside the workshops generally was more challenging for the teachers than expected (see figure 5.1).

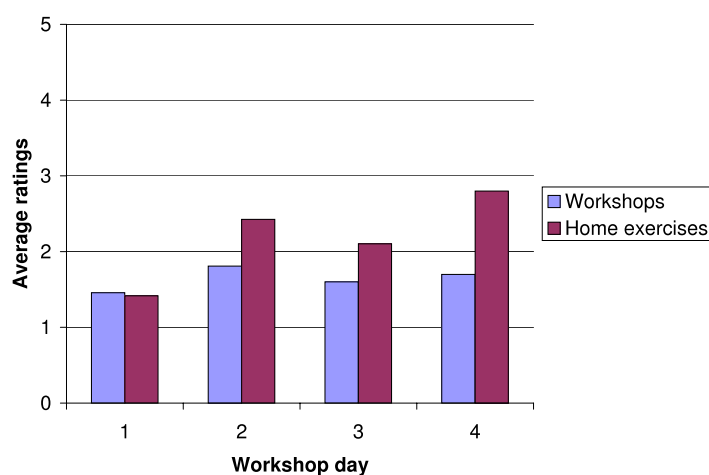


Figure 5.1: Average workshop and home exercise ratings

A Wilcoxon test used to statistically compare the average rating of the workshop activities and homework exercise of each day seems to support these theories. The test revealed significant differences on three out of four days (see table 5.11) on which the home exercise ratings exceeded the average difficulty ratings of the corresponding workshop activities.

	Day 1	Day 2	Day 3	Day 4
T	15.82	12.92	13.50	10.42
Z	-0.488	-2.415	-1.913	-4.472
α (one-tailed)	0.313	0.008	0.028	0.000

Table 5.11: Wilcoxon test results for workshop and home exercise ratings

However, the values for the standard deviation and therefore distribution of the data indicate that the differences found between the average ratings are not practically meaningful and could also be random results. Nevertheless, it needs to be taken into account

that some of the home exercises were rated more difficult than the corresponding workshop activities. Therefore, explanations for these additional impediments for the use of GeoGebra at home need to be found in order to facilitate independent use of GeoGebra after participation in a future introductory workshop.

5.3.2 Home Exercise Ratings

Experiences obtained during the home exercise discussions at the beginning of the workshops provided a basis for finding feasible explanations for the differences between the difficulty ratings of workshops and home exercises.

Home Exercise 1

With an average difficulty rating of $\bar{x} = 1.42$, home exercise 1 ‘Equilateral Triangle’ was rated to be of about the same difficulty level as the average workshop activity of this day ($\bar{x} = 1.46$). The participants seemed to be well prepared for this home exercise after practicing how to construct geometric figures with GeoGebra during the workshop. They seemed to be comfortable with using the properties of an equilateral triangle in order to find appropriate tools for its construction and applying the drag test in order to find potential mistakes in their construction.

Home exercise 2

Although home exercise ‘Drawing Tool to Check for Axes of Symmetry’ was a combination of two of the workshop activities ‘Drawing tool for symmetric figures’ ($\bar{x} = 1.84$) and ‘Inserting a background image’ ($\bar{x} = 1.74$), it was rated to be significantly more difficult ($\bar{x} = 2.42$) than both corresponding activities. A possible reason for these unequal difficulty ratings are major problems with finding a suitable picture on the Internet that could be inserted into GeoGebra, then used as a background image for the creation of the drawing tool to check for axes of symmetry displayed in the picture.

In order to avoid unnecessary problems concerning the use of images, additional information on how to search the Internet for suitable pictures and how to resize them prior to inserting them into GeoGebra by using appropriate software should be given during the workshop and in form of a handout. Additionally, adequate image files should be provided by the instructor in order to ensure that every participant is able to complete the exercise using GeoGebra without having to worry about finding a suitable image file on the Internet.

Home Exercise 3

Although the home exercise rating on day 3 ($\bar{x} = 2.10$) exceeds the difficulty rating of the corresponding workshop ($\bar{x} = 1.60$), it is quite similar to some of the individual activity ratings (see activities ‘Square’, ‘Rotation of a polygon’, ‘Polynomial functions’, and ‘Library of functions’ in table 5.5). Therefore, the difficulty level of the home exercise

doesn't seem to be inappropriate for the target audience, although it seemed to be rather challenging for the teachers.

Home Exercise 4

On day 4 the widest difference between the average rating of the workshop and the home exercise occurred (difference $\bar{x}_{HE} - \bar{x}_{WS} = 1.10$, see table 5.10). Many teachers seemed to have difficulties using MS Word in combination with GeoGebra in order to create the cards for a 'Function Domino' game.

Although those difficulties are not directly related to the use of GeoGebra, the design of the home exercise requires more instructions on how to use MS Word, especially how to create a table, as well as how to insert and resize an image. Those technical skills obviously cannot be assumed to be known by secondary school teachers and need to be addressed separately during the corresponding workshop or on detailed handouts.

5.3.3 Home Exercises Feedback

In addition to assigning difficulty ratings, participants had the chance to explain their problems on the home exercise surveys by means of an open ended question (see appendix, chapter B, pp. 237).

General Review of Responses

Table 5.12 lists the number of responses per home exercise as well as the average number of words used to explain '*which parts of the homework were difficult*' for the teachers. For each home exercise about half of the participants responded to the question with an average of 14.5 words per statement.

	Exercise 1	Exercise 2	Exercise 3	Exercise 4
Total surveys	34	39	40	41
Total responses	18	26	16	22
Responds in %	53%	67%	40%	54%
Total words	293	321	221	356
Words / response	16	12	14	16

Table 5.12: Home exercises feedback

In order to get an overview of the general attitude of the participants towards the home exercises, their responses were systematically analyzed based on a Grounded Theory approach [Strauss and Corbin, 1998]. 8 different keywords were assigned to the responses and they were grouped into the three categories 'Design of Home Exercises', 'GeoGebra Use at Home', and 'General Computer Use at Home' (see table 5.13). For explanations of the keywords see appendix (section C.3).

Home Exercises Feedback		
Category	Keyword	Frequency
III a Design of Home Exercises	1 Feedback	14
	2 Content	8
III b GeoGebra Use at Home	3 Use of tools	11
	4 Algebraic input	14
	5 Features	16
III c General Computer Use	6 Computer issues	9
	7 Pictures	14
	8 MS Word use	21

Table 5.13: Coding and frequencies for feedback about *Home Exercises*

Detailed Review of Responses

Design of Home Exercises: In this category the general feedback about the home exercises, as well as their contents and design is summarized.

- *Feedback:* In 14 responses teachers gave general feedback about the difficulty level of the home exercises as well as the usability of GeoGebra. While two teachers were overwhelmed with the exercises, six teachers stated that although they were not difficult, they would have needed more time to practice with GeoGebra at home. Six teachers liked to work with GeoGebra due to its ease of use and ‘*had fun playing*’ with the software.

Considering those statements, the majority of teachers seemed to have a positive attitude about using GeoGebra at home and trying out new tasks on their own.

- *Content:* 8 responses referred to the content of home exercises. Teachers stated that they didn’t know the content in the first place (e.g. properties of geometric figures) and therefore had difficulties completing the home exercise, or that it was ‘*challenging to remember all construction steps*’ necessary to complete the exercise.

Again, detailed handouts for the workshop contents could have helped teachers with finishing the home exercises. Construction protocols for the geometry exercises on the first two days probably would have especially supported the teachers and decreased the number of difficulties caused by unfamiliar mathematical content.

GeoGebra Use at Home: In this category feedback concerning the use of GeoGebra at home is summarized.

- *Use of tools:* 11 responses were related to the use of GeoGebra tools. Some teachers didn’t know which tool was appropriate for which construction step and therefore had a hard time working on some of the tasks. Individuals also

reported that they couldn't remember how to find help on how to operate a tool, and that they found certain tools challenging (e.g. *Slider*, *Polygon*).

In order to prevent difficulties with tools that were already introduced during the workshops, detailed handouts could be a solution. Additionally, the importance of reading the *Toolbar help* in order to find out how to operate a tool needs to be more emphasized.

- *Algebraic input*: In 14 responses teachers reported difficulties concerning the input of algebraic expressions and commands in GeoGebra. For example, seven teachers reported that they had difficulties using the command *Vertex* after they entered the equation of a parabola p into the input field. Apparently, some of them got confused about naming the vertex using $V = \dots$ and plugging in the name of parabola p in order to get its vertex (e.g. `Vertex[p]`).

These statements indicate that algebraic input needs to be introduced more carefully with special focus on the syntax of commands. Again, handouts with information about how to enter algebraic expressions and commands in combination with the most common error messages would come in handy for the participants.

- *Features*: 13 responses were related to features of GeoGebra the teachers found challenging. While some teachers forgot how to undo construction steps, six participants stated that they had difficulties inserting images into GeoGebra. Since they couldn't remember how to operate the *Insert image* tool, some teachers tried to copy and paste an image file into GeoGebra, which didn't work at all. Five responses dealt with tracing an object, whereby some teachers were confused about the fact that a trace can't be saved, and traces disappear whenever the graphics are refreshed by GeoGebra. Three additional responses concerned the export possibilities of GeoGebra, which the teachers found easy, especially compared to inserting the picture into an MS Word file.

Since undoing mistakes is very important especially for GeoGebra beginners, the different possibilities to delete objects and undo construction steps need to be summarized and mentioned frequently during the workshops. Also, the *Trace* feature requires additional explanations and practice in order to prevent unnecessary confusion and frustration when practicing at home.

Inserting images into GeoGebra seems to be rather challenging for the teachers, and therefore, needs special practice time during the introductory workshops using provided picture files. Additionally, the software feature itself could be modified in order to prevent some of the difficulties mentioned above in future workshops (e.g. inserting pictures using copy and paste, dragging a picture file into the graphics window with the mouse).

General Computer Use: In this category general computer problems the teachers experienced when working on their home exercises are summarized.

- *Computer issues*: 9 responses were related to general computer problems the teachers experienced at home. Apart from not being able to save files on their machines, several teachers reported that they had issues with their Internet connection.
- *Pictures*: In 14 responses teachers reported that they couldn't find suitable pictures on the Internet, or that they haven't been able to resize their images using appropriate software prior to inserting them into GeoGebra.
- *MS Word use*: A total of 21 responses dealt with the use of MS Word in order to create cards for a 'Function Domino' game. Five participants reported difficulties with inserting images into MS Word in general, while 12 teachers had major problems creating the domino cards by using a table in order to align a picture and a function equation. Some teachers also couldn't remember how to resize the image once they inserted it into the MS Word document so that it would subsequently fit on one of the domino cards.

Although the general computer and Internet problems can't be prevented by the workshop instructor, difficulties with inserting images into GeoGebra could easily be prevented by offering a selection of suitable pictures the teachers can use for their home exercise. Concerning the use of MS Word in order to create teaching materials in combination with GeoGebra, more detailed instructions on how to insert pictures, create tables, and formatting in general seem to be necessary for this target audience of secondary school teachers. Detailed assistance could also be provided in form of a handout with step-by-step instructions.

Chapter 6

Complexity Criteria for DGS Tools

In this chapter complexity criteria are established that allow determination of the general difficulty level of dynamic geometry tools. After assigning the introduced GeoGebra tools to three groups according to their difficulty ratings, the tool ratings on the day of introduction are analyzed and compared to the ratings on the day of first reuse. Common characteristics of these GeoGebra tools are summarized, providing a basis for the establishment of a set of complexity criteria for dynamic geometry tools that help to determine the general difficulty level of each GeoGebra tool. Furthermore, the complexity criteria are applied to the tools of *Cabri Geometry* and *Geometer's Sketchpad* in order to find out about their applicability and potential relevance for other dynamic geometry software packages.

6.1 Difficulty Level Groups for GeoGebra Tools

During the four GeoGebra introductory workshops a total of 21 dynamic geometry tools were introduced. Every day the participating teachers rated the difficulty of all tools used in each workshop on a scale from 0 ('very easy') to 5 ('very difficult'). Table 6.1 shows which tools were introduced in each workshop and when they were used for the second time in another workshop. Additionally, the average ratings for all tools on the days of introduction and first reuse are listed. Since not all of the tools were reused in another workshop¹, they were not rated for a second time and could not be part of the difficulty comparison.

In order to classify the introduced GeoGebra tools into three difficulty level groups 'easy to use', 'middle', and 'difficult to use', as well as to define corresponding thresholds, the difficulty ratings of tools on the day of their introduction were examined. Thereby, the critical value for 'easy to use' tools was set at $t_{easy} = 0.99$, producing an interval width of 0.51 between the easiest tool rating and the threshold for this group.

¹While in an ideal study situation all introduced tools would have been reused and rated again on another day, this wasn't possible within the context of this study. Since the series of introductory workshops was part of a professional development event, the design of the workshops had to focus more on the participants' needs and limited time for each workshop, than on collecting data for the research study.

GeoGebra Tool		Introduction		Reuse	
1	Segment between two points	WS I	0.81	WS II	0.54
2	Circle with center through point	WS I	0.86	WS II	0.67
3	Intersect two objects	WS I	1.10	WS II	0.58
4	Line through two points	WS I	1.07	WS II	0.52
5	Move	WS I	0.74	WS II	0.56
6	Polygon	WS I	1.05	WS II	0.77
7	Line bisector	WS I	1.38	–	–
8	Show / hide object	WS I	1.31	WS II	0.79
9	Move drawing pad	WS I	1.25	–	–
10	Zoom in / Zoom out	WS I	1.13	–	–
11	Perpendicular line	WS I	1.39	WS III	0.83
12	Parallel line	WS II	1.02	WS III	0.85
13	Angle	WS II	1.23	–	–
14	Mirror at line	WS II	1.17	–	–
15	New point	WS II	0.48	WS III	0.63
16	Rotate object around point by angle	WS II	1.71	–	–
17	Insert image	WS II	1.67	–	–
18	Slider	WS III	1.43	–	–
19	Slope	WS III	1.37	–	–
20	Insert text	WS III	1.46	–	–
21	Tangents	WS IV	1.27	–	–

Table 6.1: Average tool ratings on days of introduction and first reuse

Difficulty level group	Interval	Tools	Percentage
‘Easy to use’ tools	0.48 – 0.99	4	19.05%
‘Middle’ tools	1.00 – 1.35	10	47.62%
‘Difficult to use’ tools	1.36 – 1.71	7	33.33%

Table 6.2: Difficulty level groups for GeoGebra tools

Afterwards, the remaining range of width 0.72, between this threshold and the most difficult tool rating, was split into two halves in order to get the threshold of $t_{diff} = 1.36$ for the group of ‘difficult to use’ tools (see table 6.2). Thus, on the day of introduction, four tools were rated to be part of the ‘easy to use’ group (19.05%), while ten tools fell into the ‘middle’ group (47.62%), and seven tools formed the ‘difficult to use’ group (33.33%).

Even the most ‘difficult to use’ tool (*Rotate object around point by angle*, $\bar{x} = 1.71$) was rated on average within the lower third of the provided scale. Therefore, none of the introduced GeoGebra tools seems to have caused major difficulties for the participants, a fact that supports feedback related to the user friendliness of this software. Although the

differences between difficulty ratings of the introduced tools are not very serious, the terms ‘easy to use’, ‘middle’, and ‘difficult to use’ are used throughout this thesis in order to address subtle distinctions between the difficulty levels of GeoGebra’s dynamic geometry tools.

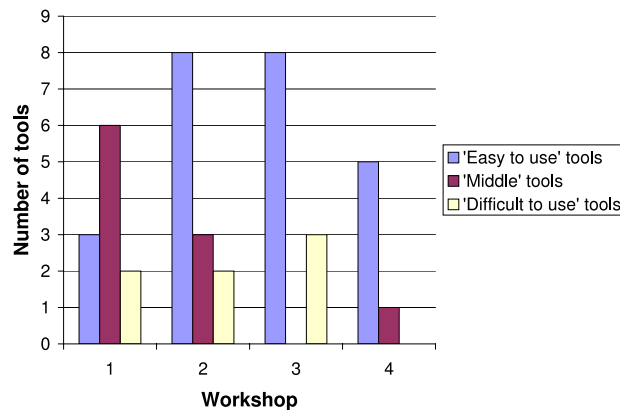


Figure 6.1: Distribution of difficulty level groups in each workshop

Figure 6.1 shows the distribution of tools used in each workshop according to their difficulty level groups. In the first workshop all eleven tools were introduced. Of those, three tools were rated ‘easy to use’, six tools were rated ‘middle’, and two tools were rated ‘difficult to use’. In the second workshop thirteen tools were used in total including six that were introduced for the first time. In this workshop, eight tools were rated ‘easy to use’, three tools were rated ‘middle’, and two tools were rated ‘difficult to use’. In the third workshop eleven tools were used including three that were introduced for the first time. In this case, eight tools were rated ‘easy to use’ and three tools were rated ‘difficult to use’. In the fourth and last workshop, six tools were used of which just one was introduced as a new tool. Here, five tools were rated ‘easy to use’ and one tool was rated ‘middle’.

In general, the percentage of tools rated to be ‘easy to use’ increased in each workshop. In workshop I just 27.3% of all tools used were rated ‘easy to use’ while this percentage more than doubled in workshop II (61.5%). On the next days, 72.7% of the tools used in workshop III seemed to be ‘easy to use’ as well as 83.3% of the tools used in workshop IV. This could refer to the rising familiarity with the software, as well as the additional instruction and practice using GeoGebra (see figure 6.2).

Table 6.3 shows the distribution of ‘easy to use’, ‘middle’, and ‘difficult to use’ tools that were used during the four GeoGebra workshops. All tools that were reused in one of the workshops were rated ‘easy to use’, although some of them were rated ‘middle’ or even ‘difficult to use’ on the day of their introduction.

Overall, about half of the tools introduced in one of the GeoGebra workshops were reused on another day and therefore, rated for a second time. Figure 6.3 shows both

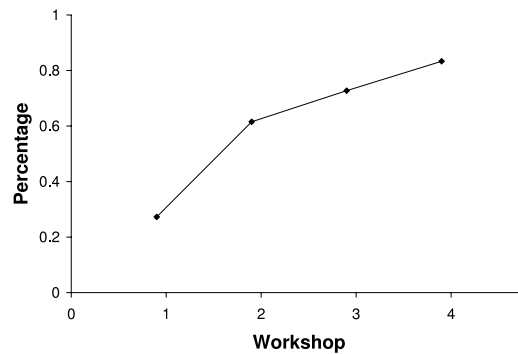


Figure 6.2: Percentages of 'easy to use' tools

	Total	'Easy to use'	'Middle'	'Difficult to use'
Workshop I				
Introduced	11	3	6	2
Reused	0	0	0	0
Workshop II				
Introduced	6	1	3	2
Reused	7	7	0	0
Workshop III				
Introduced	3	0	0	3
Reused	8	8	0	0
Workshop IV				
Introduced	1	0	1	0
Reused	5	5	0	0

Table 6.3: Number of tools in each difficulty level group

average ratings for each of those ten tools, as well as the thresholds for 'easy to use' and 'difficult to use' tools (for tool numbers and corresponding tool names see table 6.1).

Four tools that were rated 'easy to use' on the day of their introduction were reused on another day, namely tools *Segment between two points* (nr. 1), *Circle with center through point* (nr. 2), *Move* (nr. 5), and *New point* (nr. 15). All of those tools were still rated 'easy to use' when used in another workshop, whereby three of them even got a lower difficulty rating on the second rating day. Only the *New point* tool received a higher difficulty rating when it was reused in workshop IV. A possible reason for this is that teachers mixed up creating a new point by typing its coordinates into the input field, which is obviously more complex than clicking once on the drawing pad. Both ways of creating a new point were used within activity *Coordinates of points* and could therefore have been difficult to separate during the workshop rating later on.

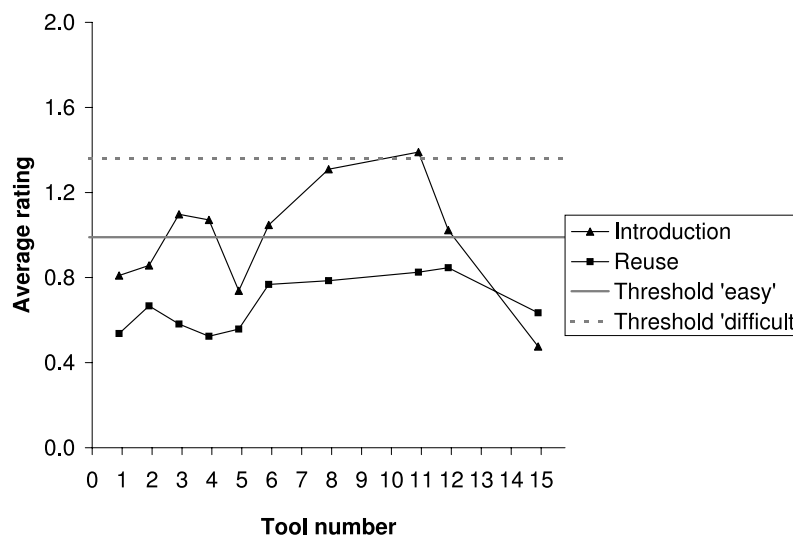


Figure 6.3: Comparison of tools ratings on days of introduction and reuse

Five tools that were initially rated to be part of the ‘middle’ group were also rated for a second time, namely the tools *Intersect two objects* (nr. 3), *Line through two points* (nr. 4), *Polygon* (nr. 6), *Show / hide object* (nr. 8), and *Parallel line* (nr. 12). Again, all of these tools were rated ‘easy to use’ on the day of reuse. The differences between the particular difficulty ratings of tools 3, 4, and 8 average about 0.5, lowering their initial rating by about 45%.

Only one tool that was rated ‘difficult to use’ on the day of its introduction, namely the *Perpendicular line* tool (nr. 11), was reused in another workshop and therefore rated for a second time. Its difficulty rating decreased by 40%, also placing the tool in the ‘easy to use’ group when reused on another day.

Since all tools that were reused on another day were rated to be ‘easy to use’ in their second rating, the conclusion was drawn that additional instruction on how to use GeoGebra as well as more practice time during the workshops and at home, help teachers to get comfortable with using dynamic geometry tools irrespective of their initial difficulty ratings.

6.2 Description of Introduced GeoGebra Tools

After assigning the introduced GeoGebra tools to their subjective difficulty level groups, the use of those tools was analyzed in order to find common characteristics that would help to generally identify ‘easy to use’ or ‘difficult to use’ GeoGebra tools. Figure 6.4 shows the introduced GeoGebra tools arranged by difficulty levels. Within each difficulty level group, the tools are organized according to their order of introduction.

In the following sections, each of the 21 introduced tools as well as their functionality are described, in order to provide a basis for the following complexity analysis of GeoGebra tools and the establishment of complexity criteria for dynamic geometry tools in general.

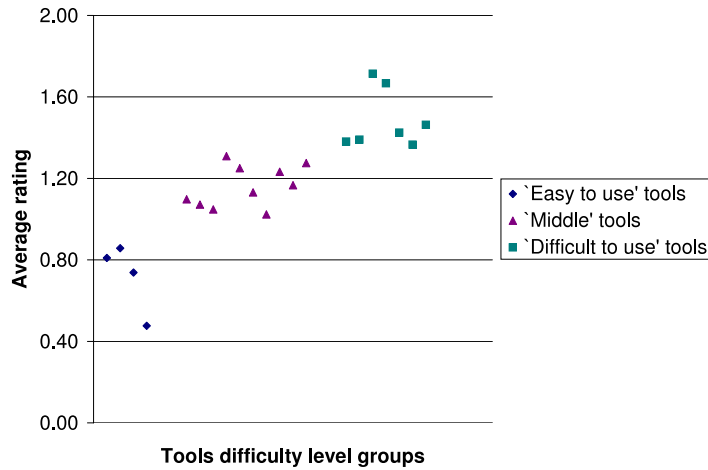


Figure 6.4: Difficulty level groups of tools

6.2.1 'Easy to use' Tools Group

Table 6.4 shows those GeoGebra tools that were rated 'easy to use' on the day of their introduction, organized by their subjective difficulty ratings. It also displays their average ratings which range between 0.48 and 0.86 with a mean of $\bar{x} = 0.72$ and a standard deviation of $s = 0.97$.

Tool	Introduced	Mean
1 New point	WS II	0.48
2 Move	WS I	0.74
3 Segment between two points	WS I	0.81
4 Circle with center through point	WS I	0.86

Table 6.4: Difficulty level group of 'easy to use' GeoGebra tools

New point

The *New point* tool was rated the easiest tool of all introduced GeoGebra tools ($\bar{x} = 0.48$, $s = 0.77$). It is the default tool of the *Point Toolbox* and requires just a click on the drawing pad in order to create a new point at the selected position.

In survey II participants also got the chance to rate the difficulty of using this tool in order to create a point on an already existing object. Although this task can be a little tricky for beginners, the average rating was $\bar{x} = 0.68$ with a standard deviation of $s = 1.14$. Therefore, participants didn't seem to have problems using the *New point* tool at all, neither to create a 'usual' new point, nor a new point on an object.

Move

The *Move* tool was rated as the second easiest of all tools ($\bar{x} = 0.74$, $s = 1.01$). It is the default tool of the *Move Toolbox* and just needs an existing object for its application. In order to operate the *Move* tool, a user needs to click on any free object, hold the mouse key pressed, and drag the object to another position.

Segment between two points

The *Segment between two points* tool was rated third of all 'easy to use' tools ($\bar{x} = 0.81$, $s = 1.04$). It is part of the *Basic Lines Toolbox*, but is not the default tool. In order to operate this tool, a user needs to click twice, either on the drawing pad, or on already existing points, in order to specify the endpoints of the segment. The order of clicks is irrelevant.

Circle with center through point

The *Circle with center through point* tool was the last tool rated 'easy to use' by the participants ($\bar{x} = 0.86$, $s = 1.03$). It is the default tool of the *Circle Toolbox* and requires two clicks to be operated. It doesn't matter if a user clicks on the empty drawing pad or on already existing points. However, the first click specifies the center point of the circle, while the second click defines a point that lies on the circle and therefore determines its radius. Thus, the order of clicks is relevant for this tool.

6.2.2 'Middle' Tools Group

Table 6.5 lists those GeoGebra tools rated 'middle' on the day of their introduction as well as their average ratings organized according to their subjective difficulty ratings. These range between 1.02 and 1.31. The mean of all 'middle' tools ratings is $\bar{x} = 1.16$ with a standard deviation of $s = 1.16$.

Parallel line

The *Parallel line* tool is the first of the 'middle' tools ($\bar{x} = 1.02$, $s = 1.03$). It is not the default tool, but can be found in the *Special Lines Toolbox*. This tool requires two clicks to be applied, but just one of those clicks needs to be on an existing object (e.g. line or segment). The other click refers to an existing point or an empty spot on the drawing pad.

	Tool	Introduced	Mean
1	Parallel line	WS II	1.02
2	Polygon	WS I	1.05
3	Line through two points	WS I	1.07
4	Intersect two objects	WS I	1.10
5	Zoom in / Zoom out	WS I	1.13
6	Mirror object at line	WS II	1.17
7	Angle	WS II	1.23
8	Move drawing pad	WS I	1.25
9	Tangents	WS IV	1.28
10	Show / hide object	WS I	1.31

Table 6.5: Difficulty level group of ‘middle’ GeoGebra tools

The order of clicks is irrelevant for this tool, but two different types of objects are involved (e.g. point – line or point – segment).

Polygon

The *Polygon* tool was rated to be second of the ‘middle’ tools ($\bar{x} = 1.05$, $s = 1.25$) and is the default tool of the *Polygon Toolbox*. Depending on the number of vertices of the desired polygon, a user needs to click several times, either on already existing points or on the empty drawing pad, in order to define the vertices of the polygon. The order of clicks determines the shape of the polygon. The challenge of operating this tool is to click on the first vertex for a second time after selecting all other vertices in order to close the polygon.

Line through two points

The *Line through two points* tool is the third tool of the ‘middle’ group ($\bar{x} = 1.07$, $s = 1.00$). It is the default tool of the *Basic Lines Toolbox* and requires two clicks to be applied. However, a user can click on already existing points or on the drawing pad in order to create a line through these points. The order of clicks is irrelevant.

Intersect two objects

The *Intersect two objects* tool was rated fourth of the ‘middle’ tools ($\bar{x} = 1.10$, $s = 1.02$). It is not the default tool, but can be found in the *Point Toolbox*. This tool requires two already existing objects (e.g. circle and line) in order to be operated and can be applied in two ways. On the one hand, a user can click directly on the intersection point of those two existing objects. This creates the one intersection point closest to the position of the click. On the other hand, a user can successively click on the two objects which creates all intersection points (if there are more than one).

Zoom in / Zoom out

Tools *Zoom in* and *Zoom out* were rated in fifth place of the ‘middle’ tools group ($\bar{x} = 1.13$, $s = 1.21$). They are not default tools, but can be selected from the *General Tools Toolbox*. In order to ‘zoom in’ or ‘zoom out’ a user needs to click on the drawing pad in order to define the center of zooming. When using these tools, no objects are involved directly, but all objects on the drawing pad are affected.

Mirror object at line

The *Mirror object at line* tool is the sixth of the ‘middle’ tools ($\bar{x} = 1.17$, $s = 1.08$). It is not the default tool, but part of the *Transformation Toolbox* and needs to be applied to two already existing objects (e.g. polygon – line). A user must click on the object to be mirrored before clicking on the line, thereby specifying it as the line of reflection. Thus, the order of clicks is relevant for this tool.

Angle

The *Angle* tool is the seventh tool of this group ($\bar{x} = 1.23$, $s = 1.27$). It is the default tool of the *Measure Toolbox* and can be used in three different ways. At first, a user can specify an angle by clicking three times on the drawing pad or on three already existing points. Here, the order of clicks is very important, because it determines if the angle is measured in a mathematically positive or negative direction. Secondly, a user can click on two intersecting segments or lines, whereby the angle is always measured counterclockwise. Thirdly, a user can simply click in the middle of a polygon in order to get all its interior angles at the same time provided that the polygon was created by clicking on its vertices in counterclockwise direction. If the polygon was created differently, the exterior angles are created.

Move drawing pad

The *Move drawing pad* tool was rated in eighth place of the ‘middle’ tools ($\bar{x} = 1.25$, $s = 1.19$). It is the default tool of the *General Tools Toolbox* and allows a user to click and drag the drawing pad in order to adjust the visible area. Although there are no objects directly involved when applying this tool, usually all objects on the drawing pad are affected.

Tangents

The *Tangents* tool is the ninth tool in the ‘middle’ group ($\bar{x} = 1.28$, $s = 1.34$). It is not the default tool but can be selected by opening the *Special Lines Toolbox* and requires clicking on an already existing object (e.g. a circle). A second click either creates or just selects a point needed in order to create the tangent(s). The order of clicks is irrelevant.

Show / hide object

The *Show / hide object* tool was rated to be last of the ‘middle’ tools group ($\bar{x} = 1.31$, $s = 1.26$). It is not the default tool, but can be selected from the *General Tools Toolbox*. This tool requires one already existing object which can be hidden by highlighting it using the *Show / hide object* tool. After activating another tool, all highlighted objects are hidden.

6.2.3 ‘Difficult to use’ Tools Group

Table 6.6 shows those GeoGebra tools classified ‘difficult to use’ on the day of their introduction as well as their average ratings. They are organized according to their subjective difficulty ratings, which ranged from 1.37 to 1.71, with a mean of $\bar{x} = 1.49$ and a standard deviation of $s = 1.34$.

Tool	Introduced	Mean
1 Slope	WS III	1.37
2 Line bisector	WS I	1.38
3 Perpendicular line	WS I	1.39
4 Slider	WS III	1.43
5 Insert text	WS III	1.46
6 Insert image	WS II	1.67
7 Rotate object around point by angle	WS II	1.71

Table 6.6: Difficulty level group ‘difficult to use’ GeoGebra tools

Slope

The *Slope* tool is the first tool of the ‘difficult to use’ group ($\bar{x} = 1.37$, $s = 1.28$). It is not the default tool, but can be found in the *Measure Toolbox*. This tool requires clicking on an already existing line in order to create a slope triangle and obtain the value of its slope in the algebra window.

Line bisector

The *Line bisector* tool was rated in second place of this group ($\bar{x} = 1.38$, $s = 1.29$). It is not the default tool, but can be selected from the *Special Lines Toolbox*. On the one hand, this tool can be used by just clicking on a segment. On the other hand, a user can also successively click on two existing points or even on two empty spaces on the drawing pad in order to create the corresponding line bisector. The order of clicks is irrelevant for this tool.

Perpendicular line

The *Perpendicular line* tool is the third tool of the ‘difficult to use’ group ($\bar{x} = 1.39$, $s = 1.18$). It is the default tool of the *Special Lines Toolbox* and requires one already existing object (e.g. line). After clicking on a line, another click either selects an existing point or creates a new one and creates the perpendicular line through this point. Again, the order of clicks is irrelevant for this tool.

Slider

The *Slider* tool was rated fourth of the ‘difficult to use’ tools ($\bar{x} = 1.43$, $s = 1.22$) and is the default tool of the *Insert Toolbox*. In order to create a slider, a user needs to click on an empty space on the drawing pad or an already existing point. This opens a dialog window where the properties of the slider can be specified (e.g. name, interval, increment).

Insert text

The *Insert text* tool is on fifth place of the ‘difficult to use’ group ($\bar{x} = 1.46$, $s = 1.25$). It is not the default tool, but can be selected from the *Insert Toolbox*. In order to insert text, a user needs to click on the drawing pad or an already existing point. Then the desired text needs to be typed into the appearing dialog window and confirmed with the ‘Enter’ key. In order to create dynamic text a user needs to click on the corresponding object which inserts its name into the input field using the appropriate syntax.

In survey II, participants were given the opportunity to rate inserting static and dynamic text as part of the GeoGebra features for a second time. Inserting text was still rated to be one of the most difficult introduced GeoGebra features (see section 7.2) with an average rating of $\bar{x} = 2.00$ ($s = 1.59$) for static text and $\bar{x} = 2.28$ ($s = 1.49$) for dynamic text. Therefore, participants should be given additional instruction and the chance to practice inserting text into the graphics window in order to help them overcome the recurring difficulties and to understand the concept behind this feature and tool.

Insert image

The *Insert image* tool was rated to be in sixth place of this group ($\bar{x} = 1.67$, $s = 1.59$). It is not the default tool, but can be selected by opening the *Insert Toolbox*. In order to insert an image into the GeoGebra graphics window, a user needs to click on the drawing pad or an existing point. This opens a dialog window where the desired image can be selected from the files saved on the computer.

Rotate object around point by angle

The *Rotate object around point by angle* tool was rated the most ‘difficult to use’ tool ($\bar{x} = 1.71$, $s = 1.53$). It is not the default tool, but can be selected from the *Transformation Toolbox*. In order to rotate an object around a point, a user needs to click on an existing

object first (e.g. polygon). Then, the center of rotation needs to be specified by either clicking on the drawing pad or on an already existing point. This opens a dialog window where the user can enter the desired rotation angle. The order of clicks is relevant for this tool.

6.3 Complexity Analysis of GeoGebra Tools

After summarizing common characteristics of the 21 GeoGebra tools introduced in the workshops, complexity criteria are established that can be applied to the dynamic geometry tools of GeoGebra (see section 6.4), as well as to other dynamic geometry software programs (see section 6.5). The goal of this complexity analysis is to be able to introduce such tools in a more effective way by considering their underlying difficulty level and trying to prevent common difficulties related to their use.

6.3.1 Common Characteristics of GeoGebra Tools Groups

After examining the descriptions of 21 GeoGebra tools that were introduced during the introductory workshops, common characteristics of tools subjectively rated to be in the same difficulty group are summarized. In order to analyze the complexity of GeoGebra tools, the following information is recorded for each tool:

- the number of actions needed to operate the tool
- whether or not the order of actions is relevant for a successful use of the tool
- the number of already existing objects required for the application of the tool
- the maximum number of different types of objects involved in the application of the tool
- if some kind of keyboard input is needed in order to use the tool
- if the tool is the default tool in its toolbox. The icon of a default tool is shown in the toolbar when starting GeoGebra.

‘Easy to use’ tools

Table 6.7 lists the four tools rated to be ‘easy to use’ on the day of their introduction as well as their means. Three of these tools are default tools in their corresponding toolbox, and therefore, they seem easier to access than other tools. Half of these tools require just one click to be applied, while the other half needs two actions. The order of clicks doesn’t matter for one of these tools, but is important for the other one.

	M e a n	D e f a u l t	A c t i o n s	O r d e r	O b j e c t s	T y p e s	I n P u t
1 New point	0.48	yes	1	–	0 or 1	1	no
2 Move	0.74	yes	1	–	1	1	no
3 Segment between two points	0.81	no	2	no	0 to 2	1	no
4 Circle with center through point	0.86	yes	2	yes	0 to 2	1	no

Table 6.7: Characteristics of ‘easy to use’ GeoGebra tools

Three of these tools require either points or no existing object at all². Therefore, they could be applied to an empty drawing pad, while the fourth tool, *Move*, obviously needs to be applied to an already existing free object of any type. All tools rated ‘easy to use’ involve one type of object at most and none of them requires keyboard input of any kind.

‘Middle’ Tools

Table 6.8 displays the ten tools rated to be in the ‘middle’ difficulty group on the day of their introduction. Only half of these tools are the default tools of their toolboxes which seems to indicate that being a default tool doesn’t influence the difficulty ratings at all, particularly because the toolbar icons change after activating another tool and the toolboxes don’t switch back to their default tools during a construction process.

In this group, the number of actions necessary varies between 1 and $n + 1$, suggesting that this is not a reasonable measure for the difficulty of GeoGebra tools. The order of clicks is relevant for half the tools that require more than one click. The number of objects involved when using these tools ranges from 0 to 3 with four tools involving two different types of objects. Three of the four tools, whose order of clicks is irrelevant, need two different types of objects in order to be successfully applied. Again, no keyboard input is required for any of these tools.

‘Difficult to use’ Tools

Table 6.9 lists the seven tools rated to be ‘difficult to use’ on the day of their introduction. Two of these tools are default tools in their toolboxes, which again seems to be a poor measure for the difficulty level of a tool. The number of actions necessary for these tools varies between one and two, whereby the order of actions matters for one of the tools with two clicks.

Both, the number of objects required in order to use these tools, as well as the number of object types involved, range from 0 to 3. Although only four out of seven ‘difficult to

²The *New point* tool can also be applied to another object (e.g. line, function) in order to create a point on this object.

		M e a n	D e f a u l t	A c t i o n s	O r d e r	O b j e c t s	T y p e s	I n P u t
1	Parallel line	1.02	no	2	no	1 or 2	2	no
2	Polygon	1.05	yes	$n + 1$	yes	≥ 0	1	no
3	Line through two points	1.07	yes	2	no	0 to 2	1	no
4	Intersect two objects	1.10	no	1 or 2	no	2	2	no
5	Zoom in / Zoom out	1.13	no	1	–	–	–	no
6	Mirror object at line	1.17	yes	2	yes	1 to 2	2	no
7	Angle	1.23	yes	1 to 3	yes	0 to 3	1	no
8	Move drawing pad	1.25	yes	1	–	–	–	no
9	Tangents	1.28	no	2	no	1 to 2	2	no
10	Show / hide object	1.31	no	2	yes	1	1	no

Table 6.8: Characteristics of ‘middle’ GeoGebra tools

use’ tools require additional keyboard input, all tools introduced that actually do require input were rated to be in this difficulty level group without exception.

6.3.2 Complexity Criteria for Dynamic Geometry Tools

The following five complexity criteria summarize common characteristics of dynamic geometry tools that are grouped according to their difficulty levels. The criteria are based on an initial difficulty level rating as well as a following analysis of GeoGebra tools that were introduced during a series of four introductory workshops. Based on these criteria, GeoGebra’s dynamic geometry tools are categorized (see section 6.4), which provides a basis for the creation of new instructional materials for introductory workshops that help to introduce dynamic geometry tools more successfully to teachers and students and prevent unnecessary difficulties known to occur during the introduction process.

Complexity Criteria for ‘Easy to Use’ Tools

Criteria 1: The tool doesn’t depend on already existing objects, or just requires existing points which can also be created ‘on the fly’ by clicking on the drawing pad. The order of actions is irrelevant and no additional keyboard input is required.

Example 1: The *Segment between two points* tool meets complexity criteria 1. By clicking twice in order to create two new points or selecting two already existing points, a segment is created between these points. It doesn’t matter which point was created / selected first.

	M e a n	D e f a u l t	A c t i o n s	O r d e r	O b j e c t s	T y p e s	I n P u t
1 Slope	1.37	no	1	–	1	1	no
2 Line bisector	1.38	no	1 to 2	no	0 to 2	1	no
3 Perpendicular line	1.39	yes	2	no	1 to 2	2	no
4 Slider	1.43	yes	1	–	0	–	yes
5 Insert text	1.46	no	1	–	0	–	yes
6 Insert image	1.67	no	1	–	0	–	yes
7 Rotate object around point by angle	1.71	no	2	yes	1 to 3	3	yes

Table 6.9: Characteristics of ‘difficult to use’ GeoGebra tools

Criteria 2: The tool directly affects only one type of existing object or all existing objects at the same time and requires just one action. Again, the order of actions is irrelevant and no additional keyboard input is required.

Example 2a: The *Move* tool meets complexity criteria 2. Any free object can be dragged with the mouse by selecting it and holding the mouse key pressed while moving the pointer. Dragging the object is the only action necessary in order to apply this tool.

Example 2b: The *Move drawing pad* tool also meets complexity criteria 2. It doesn’t directly affect a certain object, but all objects in the graphics window at the same time. Clicking on an empty spot on the drawing pad and dragging it with the mouse is the only action required for the application of this tool.

Complexity Criteria for ‘Middle’ Tools

Criteria 3: For this tool the order of actions is relevant, but no additional keyboard input is required.

Example 3: The *Polygon* tool meets complexity criteria 3. Corresponding to the desired number of vertices, several clicks are necessary to create a polygon. The order of clicks is relevant because they specify which of the vertices are going to be connected by segments.

Criteria 4: The tool requires already existing objects of the same type (except just points) or of different types. No additional keyboard input is necessary.

Example 4: The *Parallel line* tool meets complexity criteria 4. On the one hand, the tool requires selecting an already existing object in order to specify the direction of the parallel line (e.g. line or vector). On the other hand, a point needs to be selected

or created ‘on the fly’ by clicking on an empty spot on the drawing pad in order to specify the distance between the original line and the new parallel line.

Complexity Criteria for ‘Difficult to Use’ Tools

Criteria 5: The tool requires input into a dialog window and usually two or more actions whose order is relevant for a successful application.

Example 5: The *Rotate object around point by angle* tool meets complexity criteria 5. After selecting an already existing object (e.g. point, polygon), the center of rotation needs to be defined. Either an already existing point can be selected, or a new point can be created ‘on the fly’. This opens a dialog window where the desired angle, as well as the orientation of rotation can be specified. Thus, three actions, which need to be performed in a certain order, are required in order to successfully apply this tool.

6.3.3 Tools in Different Difficulty Level and Complexity Groups

Table 6.10 lists the introduced GeoGebra tools as well as their complexity group which was assigned according to the complexity criteria established earlier. Additionally, the subjectively rated difficulty level group for each tool is displayed.

A comparison of the classification of the introduced tools in complexity criteria groups and subjective difficulty level groups showed, that 75% of the tools rated ‘easy to use’ by the teachers match one of the first two complexity criteria and therefore, can be assigned to the corresponding complexity criteria group. Also, 70% of the tools rated ‘middle’ match either complexity criteria 3 or 4 and therefore, fall into the corresponding complexity group. Additionally, 57% of the tools initially rated to be ‘difficult to use’ match complexity criteria 5 and thus, are part of the group of most complex and therefore challenging tools.

An analysis of these tools assigned to a different complexity group than subjective difficulty level group revealed that there are two tools that require exactly the same actions but were rated differently. While the *Perpendicular line* tool was rated ‘difficult to use’ by the workshop participants, the *Parallel line* tool was rated ‘middle’, which matches its characterization as part of the ‘middle’ complexity group. This observation prompted a thorough analysis of the activities used to introduce those tools in order to check if the activity potentially could have an impact on the subjective difficulty rating of the tool.

When comparing the average difficulty ratings of workshop activities and tools used, a Spearman correlation test revealed a strong correlation between the workshop activities and tools ratings ($\rho = 0.894$). Figure 6.5 shows the average workshop and tools ratings for each participant as well as a trend line indicating the correlation between these two variables. Thus, participants who rated the workshop activities more difficult, also tended to rate the tools used in those activities more difficult and the other way around. This strong connection definitely needs to be taken into account when creating new instructional materials for the introduction of GeoGebra. A reduced difficulty level of the workshop activities could possibly result in facilitating the use of GeoGebra tools for novices.

Tool	Complexity Criteria	Complexity Group	Difficulty Level Group
1 New point	1	easy	easy
2 Move	2	easy	easy
3 Segment between two points	1	easy	easy
4 Line through two points	1	easy	middle
5 Zoom in / Zoom out	2	easy	middle
6 Move drawing pad	2	easy	middle
7 Slope	2	easy	difficult
8 Line bisector	2	easy	difficult
9 Parallel line	4	middle	middle
10 Polygon	3	middle	middle
11 Intersect two objects	4	middle	middle
12 Mirror object at line	3	middle	middle
13 Angle	3	middle	middle
14 Tangents	4	middle	middle
15 Show / hide object	3	middle	middle
16 Circle with center through point	3	middle	easy
17 Perpendicular line	4	middle	difficult
18 Slider	5	difficult	difficult
19 Insert text	5	difficult	difficult
20 Insert image	5	difficult	difficult
21 Rotate object around point by angle	5	difficult	difficult

Table 6.10: Complexity criteria groups for introduced GeoGebra tools

Table 6.11 lists those tools whose rated difficulty level group differs from the assigned complexity criteria group, as well as the activities used to introduce them. The following paragraphs offer reasons for these inconsistencies by reverting to experiences of workshop presenters and observations made during the evaluated GeoGebra introductory workshops.

Circle with center through point

The *Circle with center through point* tool was rated to be ‘easy to use’ by the participants after being introduced in workshop I by means of the activity ‘Constructing a line bisector with GeoGebra’. Since the tool doesn’t require any existing objects apart from two points which can also be created ‘on the fly’, it potentially could meet complexity criteria 1 (see section 6.3.2). Nevertheless, the order of clicks is relevant for this tool, because the first click specifies the center point, while the second click defines a point on the circle. Therefore, the tool meets complexity criteria 3 (see section 6.3.2) and should be considered a member of the ‘middle’ tools group.

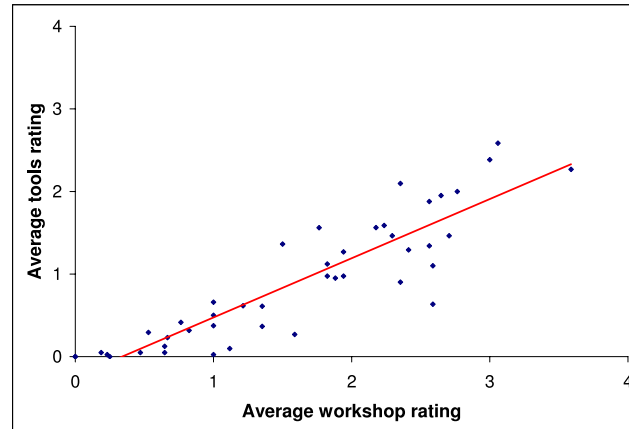


Figure 6.5: Correlation between average workshops and tools rating

Tool	Complexity Group	Difficulty Level Group	Activity
Circle with center through point	middle	easy	Line bisector with GeoGebra
Line through two points	easy	middle	Line bisector with GeoGebra
Zoom in / Zoom out	easy	middle	Line bisector with GeoGebra
Move drawing pad	easy	middle	Line bisector with GeoGebra
Slope	easy	difficult	Slope intercept form of a linear equation
Line bisector	easy	difficult	Circumscribed circle of a triangle
Perpendicular line	middle	difficult	Constructing a square

Table 6.11: Tools in different complexity and difficulty level groups

A possible reason for rating tool *Circle with center through point* to be easier than expected is that it was one of the first tools to be introduced. Additionally, the content of the corresponding activity was well-known by the teachers since they already did the actual construction of a line bisector on paper before they even started to use GeoGebra. Therefore, the teachers could focus on how to use the introduced tools, without having to struggle with the content of the activity. In addition, the first tools were introduced very carefully in order to allow the teachers to gather positive first experiences with the GeoGebra.

Line through two points

The *Line through two points* tool was introduced in the first workshop by means of the activity ‘Constructing a line bisector with GeoGebra’ and was rated to be part of the

‘middle’ group of tools by the participants. This tool doesn’t require any already existing objects apart from points which can also be created ‘on the fly’. Since the order of selecting or creating these points is not relevant, the tool meets complexity criteria 1 (see section 6.3.2) and therefore should be considered an ‘easy to use’ tool.

A closer look at the circumstances in which the *Line through two points* tool was introduced, revealed that the *Segment between two points* tool was also introduced during the same activity. This tool requires exactly the same actions to be applied as the line tool, but was rated to be in the ‘easy to use’ difficulty level group which matches its assigned complexity group. Figure 6.6 shows the ratings of these two tools for each participant. While 75% of the participants rated both tools the same, 20.5% of the teachers (9 participants) found the *Line through two points* tool more difficult to use than the *Segment between two points* tool.

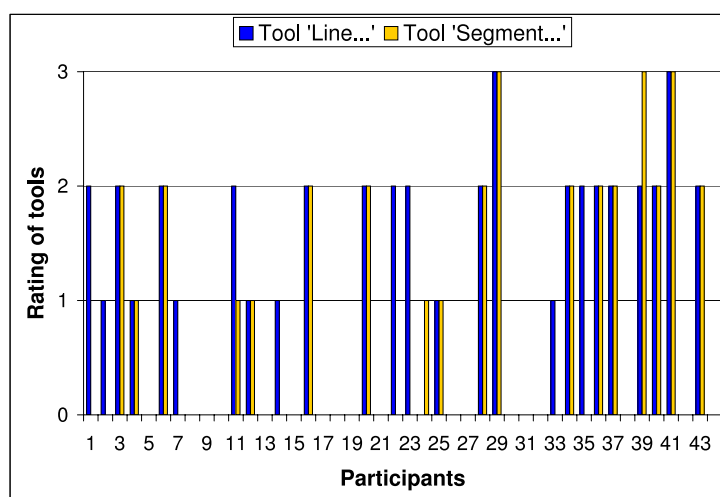


Figure 6.6: Ratings of tools ‘Line through two points’ and ‘Segment between two points’

A possible reason for this discrepancy is that the number of introduced tools in this activity may have been overwhelming for the participants. The *Segment between two points* tool was the first tool, while *Line through two points* was the last of four tools to be introduced in the activity ‘Constructing a line bisector with GeoGebra’. In between, there were two more tools introduced, namely the *Circle with center through point* tool and the *Intersect two objects* tool.

Thus, the number of tools introduced in this activity could have overwhelmed the participants and therefore may have been responsible for the subjective difficulty rating of the *Line through two points* tool. An introduction of these two tools prior to the activity of constructing a line bisector could have helped the participants to better understand the similarities between them. Thereby, creating a segment and a line without having other objects on the drawing pad could have clarified how to use these tools and emphasized their common characteristics. Additionally, a short exploration session could be offered prior to the first GeoGebra activity in a future workshop for novices, where they can try

out a selection of tools on their own and play around with them without having to worry about mathematical content and ‘doing something wrong’.

Zoom in, Zoom out and Move drawing pad

The tools *Zoom in*, *Zoom out* and *Move drawing pad* are part of the *General Tools Toolbox*. They were all introduced during the activity ‘Constructing a line bisector with GeoGebra’ in workshop I and rated to be part of the ‘middle’ difficulty level group. Since these tools can be applied in just one step and don’t have to be applied to an already existing object, they meet complexity criteria 2 (see section 6.3.2) and therefore, should be part of the ‘easy to use’ tools group.

A possible explanation for rating these tools more difficult than expected is that they were introduced along the way in order to adjust the visible part of the drawing pad. They were just a means to an end for the instructor, whereby most participants just watched them being used but didn’t try them out themselves since it wasn’t necessary for their computer screens. Therefore, those tools subjectively seemed to be more difficult and were rated accordingly. Since the group of ‘General Tools’ is a special group of tools, a separate introduction focusing on how to apply these tools should be considered, instead of just introducing them along the way.

Slope

The *Slope* tool was introduced during the third workshop by means of the activity ‘Entering the slope intercept form of a linear equation’. Since it just requires an existing line as well as one action, the tool meets complexity criteria 2 (see section 6.3.2) and therefore could be considered an ‘easy to use’ tool. Nevertheless, the *Slope* tool initially was rated ‘difficult to use’ by the teachers.

Since a Spearman correlation test revealed a strong correlation between the *Slope* tool and its introductory activity ($\rho = 0.759$), the design of this activity was given a closer look. Two factors increased the complexity of this introductory activity and, therefore, might have influenced the initial rating of the *Slope* tool. On the one hand, activity ‘Entering the slope intercept form of a linear equation’ was the very first one to combine the introduction of dynamic geometry tools with algebraic input. On the other hand, the rather advanced feature *Redefine* was introduced in this activity as well (see section 7.2.2), and it could have influenced the difficulty rating of the new tool. Therefore, the design of this activity should be modified in order to prevent unnecessary complications when introducing the *Slope* tool. Again, a prior introduction of the tool separated from the task could facilitate its introduction by allowing the teachers to focus on the use of the new tool instead of dealing with mathematical content as well.

Line bisector

The *Line bisector* tool was introduced during the first workshop by means of constructing the ‘Circumscribed circle of a triangle’. Since the tool requires just one click on an already

existing segment or two points that could also be created ‘on the fly’, it meets complexity criteria 2 (see section 6.3.2) and, therefore, could be considered an ‘easy to use’ tool. Nevertheless, the *Line bisector* tool was rated ‘difficult to use’ on the day of its introduction. Again, a strong correlation between the difficulty rating of the activity and the introduced tool *Line bisector* was revealed by a Spearman correlation test ($\rho = 0.859$).

A possible explanation for this unusual difficulty rating of the *Line bisector* tool is, that it was introduced at the very end of workshop I. Although the use of this tool was shown by the presenter, most participants didn’t actually try it out themselves and therefore, might have thought the tool to be more difficult to use. The assumption was made that tools which are presented but not actively used by the workshop participants seem to be more difficult to use and, therefore, they are rated accordingly. Thus, all tools introduced in a workshop should be included in the tasks and exercises so that the participants can actually try them out right away.

Perpendicular line

The *Perpendicular line* tool was introduced during the first workshop by means of the activity ‘Constructing a square over a given segment’ and was rated ‘difficult to use’ by the participants. Since the tool requires two different types of objects, namely a line and a point, it meets complexity criteria 4 (see section 6.3.2) and therefore should be considered a ‘middle’ tool. This classification is supported by the initial rating of the *Parallel line* tool, which was introduced during the second workshop and was rated to be part of the ‘middle’ tools group on its day of introduction. By comparing the ratings for these two tools, one can observe that 40.9% of the participants (18 teachers) rated the *Perpendicular line* tool more difficult than the *Parallel line* tool (see figure 6.7).

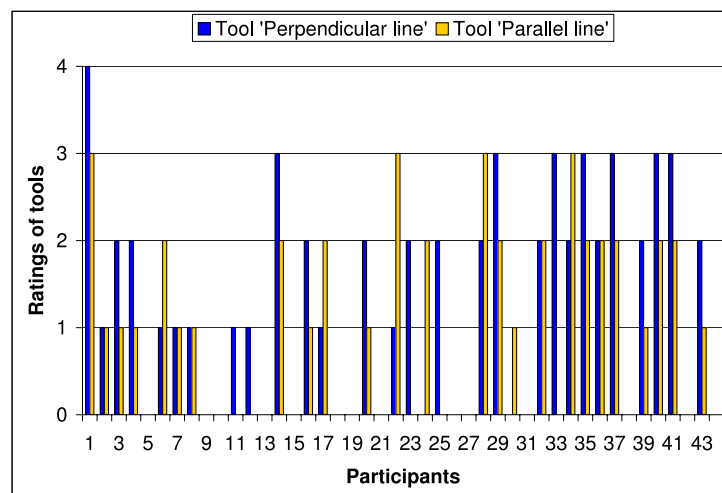


Figure 6.7: Ratings of tools ‘Perpendicular line’ and ‘Parallel line’

This result is very surprising considering the fact that both tools require exactly the same activities to be applied: The user needs to click on an already existing line and on the drawing pad (or an existing point) in order to get a parallel or perpendicular line. The order of clicks is irrelevant for both tools.

In order to find an explanation for this phenomenon the data connected with these two tools was more closely examined: In both cases a strong correlation between the difficulty ratings of the tools and their corresponding introductory activity could be found using a Spearman correlation test (the *Parallel line* tool and activity ‘Constructing a parallelogram’: $\rho = 0.731$; the *Perpendicular line* tool and activity ‘Constructing a square’: $\rho = 0.757$). Additionally, activity ‘Constructing a square over a given segment’ was rated more difficult ($\bar{x} = 1.93$) than activity ‘Constructing a parallelogram’ ($\bar{x} = 1.60$).

Based on those results, one could assume that the reason for the different ratings of these tools is connected to the different activities used to introduce them. On the one hand, the *Perpendicular line* tool was introduced by means of constructing a square which seemed to be unfamiliar to several of the teachers. Moreover, this tool was the first that involved two different types of objects (e.g. line and point). On the other hand, the *Parallel line* tool was the first tool to be introduced on the second workshop day by means of constructing a parallelogram. Although the construction process of a parallelogram might have been unfamiliar to many teachers as well, they might have benefitted from being introduced to the general construction process of quadrilaterals on the first workshop day. Being prepared for the fact of including geometric properties of the parallelogram in its construction process and being already familiar with the use of the *Perpendicular line* tool could possibly explain why the teachers rated the *Parallel line* tool to be considerably easier than the *Perpendicular line* tool.

6.4 Classification of All GeoGebra Tools

Using the complexity criteria introduced in section 6.3.2, all GeoGebra tools (currently 51 in GeoGebra 3.0) are classified and assigned to the three complexity groups ‘easy to use’, ‘middle’, and ‘difficult to use’. Based on this classification, GeoGebra’s dynamic geometry tools will be introduced in a different way during future workshops in order to make it as easy as possible for participants to get used to a new tool.

Tables 6.12 and 6.13 (pp. 128) list all GeoGebra tools, organized by toolboxes. Additionally, the tables show if the order of actions is relevant for the tools³, how many object types⁴ are involved, and if input is required for the tool⁵. Furthermore, the number and kind of actions necessary in order to apply the tools, as well as the corresponding complexity criteria and assigned complexity group are listed⁶.

³‘x’ ... ‘Order of activities is relevant’

⁴‘A’ ... ‘All existing objects are affected’

⁵‘x’ ... ‘Input is required’

⁶‘e’ ... ‘easy to use’ group; ‘m’ ... ‘middle’ group; ‘d’ ... ‘difficult to use’ group

Since some GeoGebra tools can be applied in different ways, several tools match two different complexity criteria which not always belong to the same difficulty group of tools. In these cases, the tools were assigned to the group of more complex tools in order to prevent additional impediments during the introduction of these tools.

6.4.1 ‘Easy to Use’ GeoGebra Tools

Out of currently 51 GeoGebra tools, 16 tools are part of the ‘easy to use’ complexity group. Six tools match complexity criteria 1 while ten tools are characterized by criteria 2. All but one tools of this group could uniquely be assigned to one of the two corresponding complexity criteria.

Tools Matching Complexity Criteria 1

Complexity criteria 1: *“The tool doesn’t depend on already existing objects, or just requires existing points which can also be created ‘on the fly’ by clicking on the drawing pad. The order of actions is irrelevant and no additional keyboard input is required.”* (see section 6.3.2)

New point: This tool doesn’t require any existing objects and can be used ‘on the fly’ to create a new point. However, it can also be used to create a point on another object (e.g. line, function), and therefore also meets complexity criteria 2.

Line through two points: This tool requires two already existing points which can also be created ‘on the fly’ specifying the position and direction of the line.

Segment between two points: This tool requires two already existing points which can also be created ‘on the fly’. The points represent the endpoints of the segment.

Circle through three points: This tool requires three already existing points which can also be created ‘on the fly’. The points will lie on the circle once the tool is applied.

Semicircle through two points: This tool requires two already existing points which can also be created ‘on the fly’. The points form the endpoints of the semicircle.

Conic through five points: This tool requires five already existing points which can also be created ‘on the fly’. A conic section will run through those points.

Tools Matching Complexity Criteria 2

Complexity criteria 2: *“The tool directly affects only one type of existing object or all existing objects at the same time and requires just one action. Again, the order of actions is irrelevant and no additional keyboard input is required.”* (see section 6.3.2)

Move: This tool directly affects one type of object and can be applied to every free object. It requires just one action, namely dragging the object with the mouse.

Midpoint or center: This tool directly affects one type of object (either a segment, a pair of points, or a conic section) and requires just one action which is selecting the object(s) in order to get the midpoint or center.

Line bisector: This tool directly affects one type of object (either a segment or a pair of points) and requires just one action, namely selecting an already existing segment or two points which can also be created ‘on the fly’.

Area: This tool directly affects one type of object (either a polygon, circle, or an ellipse) and requires just one action which is selecting the object in order to get its area.

Slope: This tool directly affects lines and requires just one action, namely selecting a line in order to get its slope triangle and value of slope as a dynamic text.

Move drawing pad: This tool affects all objects in the graphics window and requires one action which is dragging the drawing pad with the mouse in order to change its visible area.

Zoom in / Zoom out: These tools affect all objects in the graphics window and require one action, namely clicking on the drawing pad in order to specify the center of zoom.

Show / hide label: This tool affects one type of object at a time, but can be applied to any existing object. It requires just one action which is selecting an object in order to show or hide its label.

Delete object: This tool affects one type of object at a time, but can be applied to any existing object. It requires just one action, namely clicking on an object in order to delete it.

6.4.2 ‘Middle’ GeoGebra Tools

From the 51 GeoGebra tools, 25 are part of the ‘middle’ complexity group. Of the 25 tools in this category, 17 tools match complexity criteria 3, while eight are characterized by complexity criteria 4. Two tools of this group could also be considered ‘easy to use’ since they also match one of the first two complexity criteria for certain applications. Nevertheless, they were assigned to the ‘middle’ group in order to satisfy the more complex potential application of the tools.

Tools Matching Complexity Criteria 3

Complexity criteria 3: “*For this tool the order of actions is relevant, but no additional keyboard input is required.*” (see section 6.3.2)

Rotate around point: This tool directly affects two objects. The first object to be selected needs to be a point which represents the center of rotation. The other

object needs to be free and can be dragged with the mouse in order to rotate it around the point.

Ray through two points: This tool requires two points that can also be created ‘on the fly’. The first point selected or created represents the starting point of the ray, while the second point determines its direction.

Vector between two points: This tool requires two points which can also be created ‘on the fly’. The first point specifies the starting point of the vector, while the second point represents its endpoint.

Angle bisector: This tool requires either two lines (or segments), or three points which can also be created ‘on the fly’. Although the order of clicks doesn’t matter for lines and segments, it is relevant if the tool is applied to three points. The second point selected or created represents the vertex of the angle and lies on the angle bisector.

Locus: This tool requires two already existing points, whereby the first point to be selected needs to depend on the second point which needs to be restricted to a line. The order of clicks is very important for this tool.

Polygon: This tool requires a series of at least three points which can also be created ‘on the fly’. The order of clicks is relevant since it specifies which of the points are connected by segments forming the sides of the polygon. Additionally, the first point needs to be selected for a second time after all other vertices were specified in order to close the polygon.

Circle with center through point: This tool requires two points which can also be created ‘on the fly’. However, the first point selected or created defines the center of the circle, while the second point specifies its radius as the distance between the two points.

Circular arc with center through two points: This tool requires three points which can also be created ‘on the fly’. The first point selected or created specifies the center of the circular arc, while the second point defines its starting point as well as the radius of the arc. Although the third point defines the endpoint of the arc, it doesn’t have to lie on the arc itself.

Circumcircular arc through three points: This tool requires three already existing points which can also be created ‘on the fly’. While the first and third point selected or created form the starting point and endpoint of the circumcircular arc, the second point lies on the arc between the other two points.

Circular sector with center through two points: This tool requires three points which can also be created ‘on the fly’. The first point defines the center of the circular sector, while the other two points represent the endpoints of the arc used to create the sector.

Circumcircular sector through three points: This tool requires three already existing points which can also be created ‘on the fly’. The first point selected or created represents the starting point of the sector’s arc. The second point lies on the arc, while the third point defines the endpoint of the arc and doesn’t have to lie on the arc itself.

Angle: This tool requires either a pair of lines or segments, or three points which can also be created ‘on the fly’. In the first case, the order of selecting the lines determines if an acute or reflex angle is created. In the second case, the order of clicks determines the orientation as well as the vertex of the angle.

Mirror object at line: This tool requires an already existing line that represents the line of reflection, as well as an object to be mirrored. In order to apply this tool, the object to be mirrored needs to be selected before the line of reflection can be specified.

Mirror object at point: This tool requires an already existing object to be mirrored as well as a point which can also be created ‘on the fly’. With the first click, the object needs to be selected, while the second click defines the center of reflection.

Translate object by vector: This tool requires an object to be translated as well as an already existing vector. In order to apply the tool, the object needs to be selected prior to the vector.

Show / hide object: This tool requires an already existing object to be shown or hidden. A first click highlights the object but doesn’t hide it, unless another tool is activated.

Copy visual style: This tool requires at least two objects in order to be applied. The first object selected determines the visual style which then can be transferred to other objects by clicking on them as well.

Tools Matching Complexity Criteria 4

Complexity criteria 4: “*The tool requires already existing objects of the same type (except just points) or of different types. No additional keyboard input is necessary.*” (see section 6.3.2)

Intersect two objects: This tool requires two intersecting objects in order to be applied. The objects can be of the same type or different types. Clicking on or right next to an intersection point creates this intersection point. Selecting both objects successively creates all existing intersection points.

Vector from point: This tool requires an already existing point as well as a vector. With the first click, the starting point of the new vector is specified, while the second click attaches a copy of the initial vector to this point.

Perpendicular line: This tool requires a line, segment, or vector, as well as a point which can also be created ‘on the fly’. The order of selecting the objects is not relevant for the creation of a perpendicular line that runs through the selected point.

Parallel line: This tool requires a line, segment, or vector as well as a point that can also be created ‘on the fly’. The order of selecting the objects is not relevant for the creation of a parallel line that runs through the selected point.

Tangents: This tool requires an already existing object like a conic section or function, as well as a point. The order of selecting the two objects is irrelevant for creating a tangent to the object.

Polar or diameter line: This tool requires an already existing conic section, as well as a line or a point which can also be created ‘on the fly’. When creating the polar or diameter line of the conic section, the order of clicks is irrelevant.

Distance or length: This tool requires two already existing objects (points, lines or segments) which can also be of different types (e.g. point and line). Concerning the measurement of the length of a segment, circumference of a circle, or perimeter of a polygon, this tool also meets complexity criteria 2, because it just affects one type of object directly and can be applied in one action. Therefore, it could also be considered an ‘easy to use’ tool for some applications.

Relation between two objects: This tool requires two already existing objects of any type. By selecting both objects, a dialog window appears containing information about the relation between these two objects. The order of selecting the objects is irrelevant.

6.4.3 ‘Difficult to Use’ GeoGebra Tools

From the currently available 51 GeoGebra tools, ten are considered ‘difficult to use’ tools. They all match complexity criteria 5 and, therefore, could be uniquely assigned to this group.

Tools Matching Complexity Criteria 5

Complexity criteria 5: “*The tool requires input into a dialog window and usually two or more actions whose order is relevant for a successful application.*” (see section 6.3.2)

Segment with given length from point: This tool requires a starting point for the segment which also can be created ‘on the fly’. After selecting or creating the point, an input field appears where the length of the segment can be specified. Clicking ‘Apply’ creates the segment with given length starting at the selected point.

Regular polygon: This tool requires two points which can also be created ‘on the fly’.

By selecting or creating these two points, the side length of the regular polygon is determined and an input field appears allowing specification of the number of vertices for the polygon. Clicking ‘Apply’ creates a regular polygon with the desired number of vertices and side length.

Circle with center and radius: This tool requires a point that can also be created ‘on the fly’. By selecting or creating this point, the center of the circle is defined and an input field appears allowing specification of the radius of the circle. Clicking ‘Apply’ creates a circle with given radius.

Angle with given size: This tool requires either a segment or two points which can also be created ‘on the fly’. After selecting or creating these objects, an input field appears allowing specification of the size and orientation of the angle. Additionally, the order of creating the endpoints of the segment (or the two points) has influence on the orientation of the angle.

Rotate object around point by angle: This tool requires an object to be rotated and a center point for the rotation which can also be created ‘on the fly’. After selecting the object, the center point can be specified which opens an input field allowing entry of the size and orientation of the angle. Clicking ‘Apply’ creates the image of the given object rotated around the center point by the given angle.

Dilate object from point by factor: This tool requires an object to be dilated and a center point for the dilation which can also be created ‘on the fly’. After selecting the object, the center point can be specified, which opens an input field allowing entry of the dilation factor. Clicking ‘Apply’ creates the dilated image of the given object using the specified center point and dilation factor.

Slider: This tool doesn’t require any already existing objects. After clicking on the drawing pad, a dialog window appears allowing specification of the properties of the slider (e.g. name, interval, increment). Clicking ‘Apply’ creates the slider at the chosen position.

Checkbox to show and hide objects: This tool requires one or more object(s) to which it can be applied to. After clicking on the drawing pad, a dialog window appears allowing entry of a caption for the checkbox and selection of those objects from a list that should be influenced by the checkbox. Clicking ‘Apply’ creates a checkbox that allows the selected objects to be shown or hidden.

Insert text: This tool doesn’t require any existing objects if used to create static text. After clicking on the drawing pad, a dialog window appears allowing entry of text that should be displayed within the graphics window. Clicking ‘Apply’ creates a text box on the drawing pad. However, this tool can also be used to create dynamic text.

In this case at least one existing object is required whose name can be integrated into the text by using a special syntax.

Insert image: This tool requires a picture file to be inserted into the graphics window of GeoGebra. After clicking on the drawing pad or an already existing point, a dialog window appears allowing selection of the corresponding picture file. Clicking ‘Open’ inserts the picture at the selected position into the graphics window.

	O r d e r	T y p e s	I n P u t	#	Actions	C r i t e r i a	G r o u p
Move Toolbox							
Move		1		1	drag	2	e
Rotate around point	x	2		2	click point, drag object	3	m
Point Toolbox							
New point		1		1	click	1	e
Intersect two objects		2		1	select objects	4	m
Midpoint or center		1		1	select object(s)	2	e
Basic Lines Toolbox							
Line through two points		1		1	click (points)	1	e
Segment between two points		1		1	click (points)	1	e
Segment with given length from point	x	2	x	2	click (point), input	5	d
Ray through two points	x	1		1	click (points)	3	m
Vector between two points	x	1		1	click (points)	3	m
Vector from point		2		2	click point, select vector	4	m
Special Lines Toolbox							
Perpendicular line		2		2	click (point), select object	4	m
Parallel line		2		2	click (point), select object	4	m
Line bisector		1		1	select object(s)	2	e
Angle bisector	x	1		1	select object(s)	3	m
Tangents		2		2	click (point), select object	4	m
Polar or diameter line		2		2	click (point), select object	4	m
Locus	x	1		1	click points	3	m
Polygon Toolbox							
Polygon	x	1		2	click (points), close	3	m
Regular polygon	x	2	x	2	click (points), input	5	d
Measure Toolbox							
Angle	x	1		1	select object(s)	3	m
Angle with given size	x	2	x	2	click (points), input	5	d
Distance or length		2		1	select object(s)	4	m
Area		1		1	select object	2	e
Slope		1		1	select object	2	e

Table 6.12: Criteria classification of GeoGebra tools (part 1)

	O r d e r	T y p e s	I n P u t	#	Actions	C r i t e r i a	G r o u p
Circle Toolbox							
Circle with center through point	x	1		1	click (points)	3	m
Circle with center and radius	x	2	x	2	click (point), input	5	d
Circle through three points		1		1	click (points)	1	e
Semicircle through two points		1		1	click (points)	1	e
Circular arc with center through two points	x	1		1	click (points)	3	m
Circumcircular arc through three points	x	1		1	click (points)	3	m
Circular sector with center through two points	x	1		1	click (points)	3	m
Circumcircular sector through three points	x	1		1	click (points)	3	m
Conic through five points		1		1	click (points)	1	e
Transformation Toolbox							
Mirror object at line	x	2		1	select object(s)	3	m
Mirror object at point	x	2		2	select object, click point	3	m
Rotate object around point by angle	x	3	x	3	select object, click point, input	5	d
Translate object by vector	x	2		1	select objects	3	m
Dilate object from point by factor	x	3	x	3	select object, click point, input	5	d
Insert Toolbox							
Slider	x	1	x	2	click, dialog	5	d
Checkbox to show and hide objects	x	1	x	2	click, dialog	5	d
Insert text	x	1	x	2	click (point), dialog	5	d
Insert image	x	1	x	2	click (point), dialog	5	d
Relation between two objects		2		1	select objects	4	m
General Tools Toolbox							
Move drawing pad		A		1	drag	2	e
Zoom in		A		1	click	2	e
Zoom out		A		1	click	2	e
Show / hide object	x	1		2	select object, change tool	3	m
Show / hide label		1		1	select object	2	e
Copy visual style	x	2		1	select objects	3	m
Delete object		1		1	select object	2	e

Table 6.13: Criteria classification of GeoGebra tools (part 2)

6.5 Applicability for Other Dynamic Geometry Software

After classifying all GeoGebra tools based on the complexity criteria established before (see section 6.3.2), the tools of other dynamic geometry software are reviewed in order to check the level of general applicability of the complexity criteria and find out if they could potentially have benefits for the introduction of these software packages as well.

6.5.1 Classification of Cabri II Plus Geometry Tools

Cabri II Plus (version 1.4.1 for MS Windows) provides a total of 69 pre-defined dynamic geometry tools that can be activated over the toolbar and operated with the mouse in order to create dynamic geometric constructions. Since Cabri is pure dynamic geometry software, hardly any keyboard input is required when using these tools. Instead, required values can be extracted from a dynamic figure and reused with different tools. In general, Cabri and the DGS part of GeoGebra are very similar in that available tools are organized in toolboxes and can be accessed via the toolbar. Since Cabri doesn't provide a *Context menu* or *Properties dialog* like GeoGebra, many of these features are represented as tools as well.

In Cabri II Plus a total of 32 tools have exact equivalents in GeoGebra (version 3.0). These only differ in their names, but they can be applied in exactly the same way. Therefore, the complexity classification of these tools can be transferred one-to-one from the GeoGebra tools to the corresponding Cabri tools (see table 6.14). Out of this collection, there are 11 'easy to use', 20 'middle', and one 'difficult to use' tools in Cabri II Plus.

In addition to these equivalent tools, there are six more tools in Cabri that have matching GeoGebra tools. Although the created objects are more or less the same, the corresponding tools differ in their application and therefore, are likely to be assigned to a different complexity group than their counterparts in GeoGebra. Table 6.15 lists these Cabri tools, as well as the corresponding GeoGebra tools and assigned complexity group for both of the programs. All Cabri tools in this category either meet complexity criteria 3 or 4, and therefore are part of the 'middle' tools group. Explanations of the tools as well as reasons for assigning them to certain complexity groups can be found in the appendix (section D.1, pp. 249).

Another 15 Cabri Geometry tools correspond to GeoGebra features that can be accessed through the *Context menu* or *Properties dialog*, and thus, haven't been categorized using the complexity criteria. Nevertheless, those Cabri tools were reviewed as well, and tried to assess their complexity using the criteria already mentioned. Table 6.16 lists the Cabri tools matching this category, as well as their assigned complexity group and corresponding GeoGebra features. From this group of tools, 5 are 'easy to use', 9 tools are 'middle', and one tool is 'difficult to use'. Explanations of the tools, as well as reasons for assigning them to the corresponding complexity groups, can be found in the appendix (section D.1, pp. 251).

Cabri tool	Group	GeoGebra tool
Pointer	easy	Move
Rotate	middle	Rotate around point
Point	easy	New point
Point on Object	easy	New point
Intersection Point(s)	middle	Intersect two objects
Line	easy	Line through two points
Segment	easy	Segment between two points
Ray	middle	Ray through two points
Vector	middle	Vector between two points
Triangle	middle	Polygon
Polygon	middle	Polygon
Circle	middle	Circle with center through point
Arc	middle	Circumcircular arc through three points
Conic	easy	Conic through five points
Perpendicular Line	middle	Perpendicular line
Parallel Line	middle	Parallel line
Midpoint	easy	Midpoint or center
Perpendicular Bisector	easy	Line bisector
Angle Bisector	middle	Angle bisector
Locus	middle	Locus
Reflection	middle	Mirror object at line
Symmetry	middle	Mirror object at point
Translation	middle	Translate object by vector
Member?	middle	Relation between two objects
Distance or Length	middle	Distance or length
Area	easy	Area
Slope	easy	Slope
Angle	middle	Angle
Label	easy	Show / hide label
Text	difficult	Insert text
Mark Angle	middle	Angle
Hide / Show	middle	Show / hide object

Table 6.14: Equivalent Cabri Geometry and GeoGebra tools

Seven more Cabri tools correspond to algebraic input or the algebraic representation of objects in GeoGebra. Nevertheless, these Cabri tools are reviewed as well and their complexity is characterized using the criteria mentioned before. Table 6.17 lists the Cabri tools matching this category, as well as their assigned complexity group and the corresponding algebraic input in GeoGebra. 5 of these tools are part of the ‘middle’ tools group while the

Cabri tool	Group	GeoGebra tool	Group
Dilate	middle	Dilate object from point by factor	difficult
Regular Polygon	middle	Regular polygon	difficult
Compass	middle	Circle with center and radius	difficult
Rotation	middle	Rotate object around point by angle	difficult
Dilation	middle	Dilate object from point by factor	difficult
Hide / Show Button	middle	Checkbox to show and hide objects	difficult

Table 6.15: Similar Cabri Geometry and GeoGebra tools

Cabri tool	Group	GeoGebra feature	Access
Redefine Object	middle	Redefine	Context menu
Initial Object(s)	middle	Create new tool	<i>Tool</i> menu
Final Object(s)	middle	Create new tool	<i>Tool</i> menu
Define Macro...	difficult	Create new tool	<i>Tool</i> menu
Equation or Coordinates	easy	Show label: Value	Properties dialog
Fix / Free	easy	Fix object	Properties dialog
Trace On / Off	easy	Trace on	Context menu
Color...	middle	Color	Properties dialog
Fill...	middle	Filling	Properties dialog
Text Color...	middle	Color	Properties dialog
Thick...	middle	Line thickness	Properties dialog
Dotted...	middle	Line style	Properties dialog
Modify Appearance...	middle	–	Properties dialog
Show Axes	easy	Axes	<i>View</i> menu
Define Grid	easy	Grid	<i>View</i> menu

Table 6.16: Equivalent Cabri Geometry tools and GeoGebra features

other 2 tools meet the criteria for ‘difficult to use’ tools. Again, explanations of the tools in this category, as well as reasons for assigning them to the corresponding complexity groups, can be found in the appendix (section D.1, pp. 253).

Finally, the last 9 Cabri tools don’t have direct equivalents in GeoGebra yet, although their application can in some cases be reproduced in GeoGebra by using several tools and / or features. Table 6.18 lists the tools of this category together with their assigned complexity group. Based upon the previously determined criteria, 2 tools are ‘easy to use’, while 6 tools meet the complexity criteria for ‘middle’ tools, and one tool is ‘difficult to use’. Explanations of these tools, as well as reasons for assigning them to the corresponding complexity groups, again, can be found in the appendix (section D.1, pp. 254).

Since Cabri Geometry is very similar to GeoGebra’s dynamic geometry component, the complexity criteria for dynamic geometry tools seem to work effectively for this software

Cabri tool	Group	GeoGebra	Input example
Vector Sum	middle	Algebraic input	$\text{vector}_1 + \text{vector}_2$
Parallel?	middle	Boolean input \parallel	$\text{line}_1 \parallel \text{line}_2$
Perpendicular?	middle	Boolean input \perp	$\text{line}_1 \perp \text{line}_2$
Calculate...	difficult	Algebraic input	$3m - 2n$
Apply an Expression	difficult	Algebraic input	$P = (x(A), 2x(A))$
Numerical Edit	middle	Algebraic input	$n = 3$
Expression	difficult	Algebraic input	$f(x) = 1.5x + 3$

Table 6.17: Equivalent Cabri Geometry tools and GeoGebra algebraic input

Cabri tool	Group
Rotate and Dilate	middle
Measurement Transfer	difficult
Inverse	middle
Collinear?	easy
Equidistant?	middle
Tabulate	middle
Animation	easy
Multiple Animation...	middle
New Axes	middle

Table 6.18: Other Cabri Geometry tools

as well. As shown above, the general difficulty level of all Cabri II Plus tools can be determined using the complexity criteria for dynamic geometry tools established in section 6.3.2. Accordingly, 18 Cabri tools can be characterized as ‘easy to use’ tools, 45 Cabri tools can be assigned to the ‘middle’ tools group, and 6 Cabri tools meet the criteria for ‘difficult to use’ tools. If taken into account for the modification of already existing or the design of new introductory workshops and materials for Cabri, the complexity classification of its tools might have the potential of facilitating the first contact of novices with this dynamic geometry software.

6.5.2 Classification of Geometer’s Sketchpad Tools

Compared to GeoGebra as well as Cabri, the user interface and functionality of dynamic geometry tools is slightly different in Geometer’s Sketchpad (version 4.07D). The toolbar consists of just 10 tools, which are organized in six toolboxes. They can be used in order to create points, circles, segments, rays, or lines on the drawing pad, and they allow a user to move objects, insert text, and create custom tools.

All other provided construction features need to be selected from the menu bar and can be applied to a selection of already existing objects. This organization of construction features in Geometer's Sketchpad within the menu bar is quite different from GeoGebra and Cabri, both of which offer features that are not part of their toolbars as well. Since these features were not categorized for any of the other two programs either, just a selection of 56 Geometer's Sketchpad's construction features is reviewed. All of them are equivalent to dynamic geometry tools that are available in GeoGebra and / or Cabri.

Toolbar

From the 10 tools which are accessible using the toolbar of Geometer's Sketchpad (GSP), 4 tools can be classified as 'easy to use', while 4 others can be assigned to the 'middle' tools group, and 2 meet the complexity criteria for 'difficult to use' tools. Table 6.19 lists these ten GSP tools as well as their complexity group and their equivalents in GeoGebra. Detailed explanations of the tools, as well as reasons for their assignment to the corresponding complexity groups can be found in the appendix (section D.2, pp. 256).

GSP tool	Group	GeoGebra equivalent
Selection Arrow Tool Move	easy	Move
Selection Arrow Tool Rotate	middle	Rotate around point
Selection Arrow Tool Translate	middle	Translate object by vector
Point Tool	easy	New point
Compass Tool	middle	Circle with center through point
Straightedge Tool Segment	easy	Segment between two points
Straightedge Tool Ray	middle	Ray through two points
Straightedge Tool Line	easy	Line through two points
Text Tool	difficult	Insert text
Custom Tool	difficult	<i>Tool</i> menu

Table 6.19: Geometer's Sketchpad tools and their equivalents in GeoGebra

Construction Features

After categorizing the dynamic geometry tools of Geometer's Sketchpad's toolbar, some of its construction features, which can be accessed over the menu bar, are classified as well. The main difference compared to the use of GeoGebra and Cabri is that required objects need to be selected before such a construction feature can be activated. In order to facilitate the selection of an appropriate construction feature, only features that could successfully be applied to the actual selection of objects can potentially be selected from the menu. Nevertheless, this way of operating the software implies that the user already needs to know in advance which objects will be required for a certain construction feature. Such

knowledge usually can't be assumed in a novice and this might cause additional difficulties during the introduction of this dynamic geometry software program.

Out of nine menu items in Geometer's Sketchpad's menubar, five actually contain construction features. Since the items of the *Display* menu are very similar to the features of GeoGebra's *Properties dialog*, they are not categorized like the construction features of the other four menus.

Geometer's Sketchpad's *Construct* menu consists of 15 construction features. All of them have equivalents in GeoGebra although the names sometimes differ. Table 6.20 lists these features as well as their assigned complexity group and their equivalent GeoGebra tools. Out of this group, 4 tools are characterized as 'easy to use', while the other 11 tools can be assigned to the 'middle' tools group. Explanations of the construction features, as well as reasons for assigning them to the corresponding complexity groups can be found in the appendix (section D.2, pp. 258).

Features of the <i>Construct</i> menu		
GSP feature	Group	GeoGebra equivalent
Point on Object	easy	New point
Midpoint	easy	Midpoint or center
Intersection	middle	Intersect two objects
Segment	easy	Segment between two points
Ray	middle	Ray through two points
Line	easy	Line through two points
Parallel Line	middle	Parallel line
Perpendicular Line	middle	Perpendicular line
Angle Bisector	middle	Angle bisector
Circle By Center + Point	middle	Circle with center through point
Circle By Center + Radius	middle	Circle with center and radius
Arc On Circle	middle	Circular arc with center through two points
Arc Through 3 Points	middle	Circumcircular arc through three points
Interior	middle	Polygon
Locus	middle	Locus

Table 6.20: GSP construction features and their equivalents in GeoGebra

The *Transform* menu of Geometer's Sketchpad contains 11 construction features that can be used for different transformations. The first six tools (*Mark...*) have no equivalents in GeoGebra and can be used to specify measurements for the application of different transformations. Table 6.21 lists these features as well as their assigned complexity group and their equivalents in GeoGebra. Out of these tools, 3 tools can be characterized as 'easy to use', while 3 other tools are assigned to the 'middle' tools group, and 5 tools are classified as 'difficult to use'. Detailed explanations of the construction features, as well as reasons for assigning them to the corresponding complexity groups can be found in the appendix (section D.2, pp. 260).

Features of the <i>Transform</i> menu		
GSP feature	Group	GeoGebra equivalent
Mark Center	easy	–
Mark Mirror	easy	–
Mark Angle	middle	–
Mark Ratio	middle	–
Mark Vector	middle	–
Mark Distance	easy	–
Translate...	difficult	Translate object by vector
Rotate...	difficult	Rotate object around point by angle
Dilate...	difficult	Dilate object from point by factor
Reflect...	difficult	Mirror object at line
Iterate...	difficult	Command <code>Iterate[...]</code>

Table 6.21: GSP construction features and their equivalents in GeoGebra

Geometer’s Sketchpad’s *Measure* menu consists of 17 construction features that allow users to measure different parameters of existing objects. All of them are either similar to GeoGebra tools and features, or can be reproduced using the input field. Table 6.22 lists these features, as well as their assigned complexity group and their equivalents in GeoGebra. In this case, 11 tools are characterized as ‘easy to use’, while 5 other tools can be assigned to the ‘middle’ tools group, and 1 tool meets the complexity criteria for ‘difficult to use’ tools. Detailed explanations of the construction features, as well as reasons for assigning them to the corresponding complexity groups can be found in the appendix (section D.2, pp. 261).

The *Graph* menu of Geometer’s Sketchpad contains 13 more features that allow users to deal with analytic geometry and functions. Most of these features have equivalents in GeoGebra, or can be reproduced using the input field. Table 6.23 lists these features as well as their assigned complexity group and their equivalents in GeoGebra. Out of this group, 6 tools are characterized as ‘easy to use’, while 1 tool meets the complexity criteria for the ‘middle’ tools group, and 6 tools are assigned to the ‘difficult to use’ tools group. Detailed explanations of the construction features, as well as reasons for assigning them to the corresponding complexity groups can be found in the appendix (section D.2, pp. 263).

As shown above, all reviewed tools and construction features of Geometer’s Sketchpad can be categorized using the complexity criteria for dynamic geometry tools established in section 6.3.2. Accordingly, 28 tools and construction features can be characterized as ‘easy to use’, 24 of them can be assigned to the ‘middle’ group, and 14 tools and features meet the criteria for ‘difficult to use’ tools. Although the basic use of Geometer’s Sketchpad definitely differs from the functionality of GeoGebra and Cabri, the complexity criteria for dynamic geometry tools established before can also effectively be applied to the tools and construction features of this dynamic geometry software. This knowledge about the general difficulty level of tools and construction features might have the potential of

Features of the <i>Measure</i> menu		
GSP feature	Group	GeoGebra equivalent
Length	easy	Distance or length
Distance	middle	Distance or length
Perimeter	easy	Distance or length
Circumference	easy	Distance or length
Angle	middle	Angle
Area	easy	Area
Arc Angle	middle	Angle
Arc Length	middle	Distance or length
Radius	easy	Distance or length
Ratio	middle	Input field
Calculate	difficult	Input field
Coordinates	easy	Algebra window
Abscissa (x)	easy	Input $x(\text{Point})$
Ordinate (y)	easy	Input $y(\text{Point})$
Coordinate Distance	easy	Input $\text{abs}(x(P_1) - x(P_2))$
Slope	easy	Slope
Equation	easy	Algebra window

Table 6.22: GSP construction features and their equivalents in GeoGebra

Features of the <i>Graph</i> menu		
GSP feature	Group	GeoGebra equivalent
Define Origin	easy	–
Mark Coordinate System	easy	Axes, <i>View</i> menu
Grid Form	easy	–
Show / Hide Grid	easy	Grid, <i>View</i> menu
Snap Points	easy	Point capturing, <i>Options</i> menu
Plot Points...	difficult	Input $A = (1, 3)$
New Parameter...	difficult	Input $n = 2.5$
New Function...	difficult	Input $f(x) = 0.3x - 1$
Plot New Function...	difficult	Input $f(x) = 2x^2 + 0.4x$
Derivative	easy	Input $\text{Derivative}[\text{function}]$
Tabulate	middle	–
Add Table Data...	difficult	–
Remove Table Data...	difficult	–

Table 6.23: GSP construction features and their equivalents in GeoGebra

facilitating the first contact of novices with the Geometer's Sketchpad if taken into account for modifications in the design of introductory workshops and accompanying materials.

Chapter 7

Analysis of Other Workshop Components

This chapter focuses on other components of the evaluated GeoGebra introductory workshops, including algebraic input, commands, and GeoGebra features, and their potential impact on the difficulty ratings of the workshop activities as well as GeoGebra tools. Additionally, the role external variables, such as age or computer skills of participants, play in the introductory process of dynamic mathematics software is examined and the helper reports are summarized.

7.1 Algebraic Input and Commands Rating

Unlike pure dynamic geometry software, GeoGebra also supports the input of algebraic expressions, as well as the use of a wide range of pre-defined commands. Therefore, introductory workshops III and IV were designed to introduce the algebra window as well as the input field of GeoGebra and familiarize participants with these extensions of the software. Again, the participating teachers rated the difficulty of workshop activities and home exercises on a scale from 0 ('very easy') to 5 ('very difficult').

In order to determine if activities including algebraic input or commands were rated more difficult than pure dynamic geometry activities, the average difficulty ratings of activities of the first two and the last two introductory workshops were checked. No differences between the difficulty ratings were found between workshops that included algebraic input and commands (workshops III and IV: $\bar{x} = 1.65$, $s = 1.27$) and the ones that consisted of pure dynamic geometry activities (workshops I and II: $\bar{x} = 1.64$, $s = 1.36$).

Nevertheless, the average difficulty ratings of the corresponding home exercises were checked as well. While home exercises 1 and 2, which were pure dynamic geometry tasks, had an average rating of $\bar{x} = 1.91$ with a standard deviation of $s = 1.46$, the other two home exercises, which included algebraic input as well as the use of commands, were rated to be more difficult with an average difficulty of $\bar{x} = 2.46$ and standard deviation $s = 1.38$. Although this difference was confirmed by a Wilcoxon test on a significance level

of $\alpha = 0.002$ (one-tailed, $T = 15.30$, $Z = -2.854$), a closer look at the distribution of the data revealed that it is rather widely spread and shows rather big values for the standard deviations. Although the differences were statistically significant, the magnitude of those differences were not large enough to make a practical difference.

Since the home exercises including algebraic input and commands were rated more difficult than the pure dynamic geometry exercises, the average time spent by the participants working on the tasks was taken into account as well. With an average time of 21 minutes per home exercise, the participants spent considerably more time working on the last two home exercises than on the first two which took them only 14 minutes on average. Since all four home exercises were initially designed to require about the same amount of time, this also indicates that participants were more challenged with home exercises 3 and 4.

Finally, the average difficulty ratings of all tools on the day of introduction were compared ($\bar{x} = 1.18$, $s = 1.22$) to the average ratings of all commands used ($\bar{x} = 1.75$, $s = 1.22$). A Wilcoxon test revealed a significant difference on an $\alpha = 0.001$ level for these average ratings (one-tailed, $T = 13.67$, $Z = -3.307$), suggesting that commands were much more difficult to use than tools when introduced to the participants for the first time. Nevertheless, the distribution of the data dilutes this result, meaning that due to the rather big values for the standard deviations the magnitudes of those differences, again, were not large enough to make a practical difference.

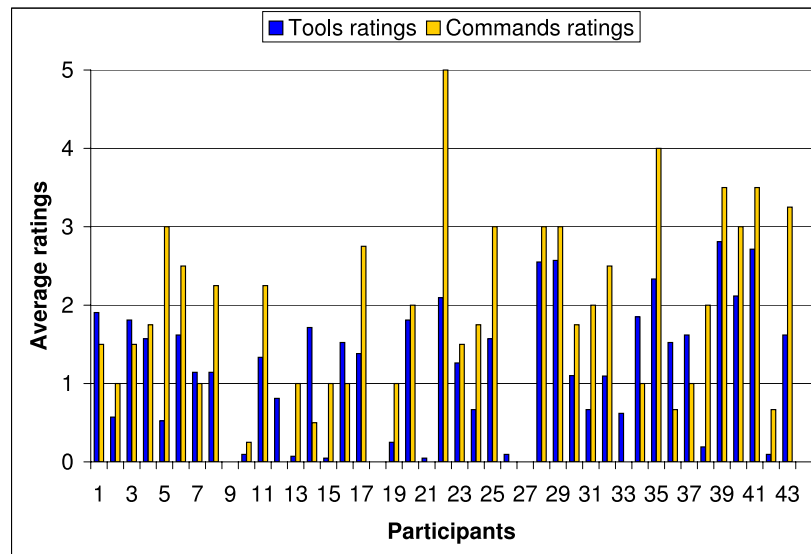


Figure 7.1: Average ratings of tools and commands on day of their introduction

Figure 7.1 shows the average difficulty ratings for tools and commands on the day of their introduction for all participants. 61.9% of the teachers rated the use of new commands more difficult than the use of new tools, compared to 33.3% of participants who found commands easier to use than tools. The use of commands definitely was more

challenging for many participants and therefore should be more carefully introduced in future GeoGebra workshops.

Although it didn't seem to be particularly difficult for the participants to follow the presenter when using algebraic input and commands during the workshops, these actions proved to be a source of problems when carried out at home. As already mentioned in section 5.3.3, participants reported that they couldn't remember the syntax necessary to enter equations and commands. They mixed up the syntax both for naming a new object (e.g. `V = . . .`) and for using the names of already existing objects within the command syntax (e.g. `Vertex[p]`).

Considering the participants' feedback, more time needs to be spent on algebraic input and commands during a GeoGebra introductory workshop, especially for explanations about the syntax. Additionally, detailed handouts containing several examples for different algebraic expressions and a list of frequently occurring mistakes (including the text of corresponding error messages) could be helpful for the participants. Concerning the use of commands, more practice time should be offered during the workshops, allowing participants to try out commands and get used to their syntax while having the possibility to ask the instructor or helpers if problems occur.

7.2 GeoGebra Features Rating

In Survey II workshop participants had the chance to rate eleven introduced GeoGebra features on a scale from 0 ('very easy') to 5 ('very difficult'). These GeoGebra features can be accessed and applied in the following ways:

- Settings can be changed using the *Menubar*.
- Features can be applied to objects by accessing the *Context menu*.
- Properties of objects can be changed in the *Properties dialog*.

Table 7.1 lists these features and gives information on how they can be accessed. Additionally, it shows in which of the four workshops the features were introduced and lists their average difficulty ratings and standard deviations. After analyzing the difficulty ratings of the different features, they were arranged into three difficulty groups using thresholds that lie within wider gaps of means. Thus, the threshold for 'easy' features was chosen to be $t_{easy} = 1.20$ and the threshold for 'difficult' features to be $t_{diff} = 1.63$, which creates about the same interval width for each group.

7.2.1 Menubar Features

GeoGebra's *Menu bar* currently (GeoGebra 3.0) consists of seven items, namely *File*, *Edit*, *View*, *Options*, *Tools*, *Window*, and *Help*. Each of these items can be selected directly and provides a submenu allowing the access to certain features as well as the change of settings.

	Mean	StDev	Access	WS
‘Easy’ features group (range: 0.68 to 1.20)				
1 Grid	0.68	0.88	Menu bar	II
2 Navigation bar	1.08	1.13	Menu bar	I
3 Construction protocol	1.17	1.30	Menu bar	I
4 Properties dialog	1.20	1.22	Menu bar or Context menu	II
‘Middle’ features group (range: 1.21 to 1.62)				
5 Point capturing	1.28	1.24	Menu bar	II
6 Trace on	1.29	1.21	Context menu	II
7 Rename	1.30	1.42	Context menu	I
8 Label types	1.35	1.25	Properties dialog	III
‘Difficult’ features group (range: 1.63 to 2.05)				
9 Background image	1.63	1.32	Properties dialog	II
10 Redefine	1.77	1.49	Context menu	III
11 Auxiliary objects	2.05	1.51	Properties dialog	III

Table 7.1: Introduced GeoGebra features arranged by difficulty ratings

Navigation Bar

In order to redo a construction in a step-by-step manner GeoGebra offers a feature called *Navigation bar for construction steps*. It was ranked second within the ‘easy’ feature group with an average rating of $\bar{x} = 1.08$ and a standard deviation of $s = 1.13$. After selecting the feature in the *View* menu, the navigation bar is displayed at the bottom of the graphics window. It contains buttons that allow the replay of a prepared construction either by clicking manually for every construction step or by using the provided ‘Play’ function (see figure 7.2).

Construction Protocol

The GeoGebra *Construction protocol* was ranked third within the ‘easy’ features group with an average rating of $\bar{x} = 1.17$ and a standard deviation of $s = 1.30$. After selecting the features from the *View* menu, the interactive table appears in a separate window. The *Construction protocol* displays all construction steps and allows redisplay of a prepared construction in a step-by-step manner using the navigation bar at the bottom of the window (see Figure 7.3). Furthermore, additional construction steps can be inserted at any position and their order can be changed as long as the relations between dependant objects are not violated.

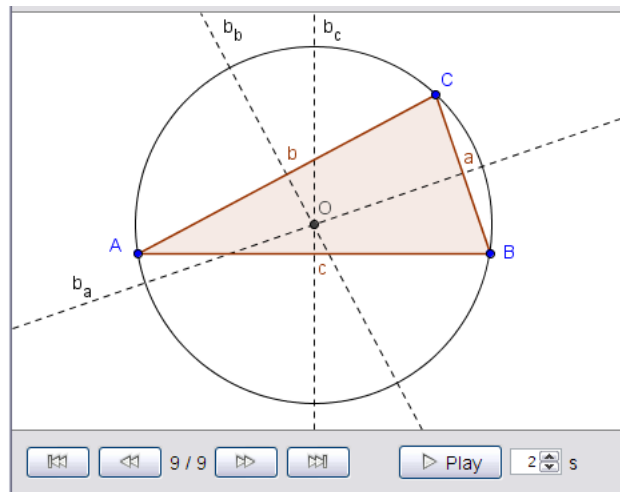


Figure 7.2: Navigation bar for construction steps

Grid and Point Capturing

Feature *Grid* was rated to be the easiest of all introduced features with an average rating of $\bar{x} = 0.68$ and a standard deviation of $s = 0.88$. After checking the feature in the *View* menu, a grid is displayed on the GeoGebra drawing pad which makes it easier to create points with integer coordinates. In order to hide the grid, item *Grid* needs to be unchecked in the *View menu*.

By contrast, the *Point capturing* feature was rated to be the first feature of the ‘middle’ group with an average rating of $\bar{x} = 1.28$ and a standard deviation of $s = 1.24$. After selecting the feature in the *Options* menu, a user can change its setting according to the following options.

- *Automatic*: This is the default setting for feature *Point capturing*. If a new point is placed close to a grid point, it is automatically positioned on integer coordinates.
- *On*: Same as option ‘Automatic’.
- *On (Grid)*: All new points are placed on the closest grid point and therefore have integer coordinates.
- *Off*: Coordinates of new points are set by the actual position of the pointer. Hereby, it is sometimes difficult to create a point with integer coordinates.

A Wilcoxon test revealed a significant difference on an $\alpha = 0.001$ level between the difficulty ratings of these two features (one-tailed, $T = 0$, $Z = -3.360$), which again is diluted by the actual distribution of the data and doesn’t make a practical difference. Nevertheless, possible reasons for the different difficulty ratings need to be found.

Although both of these features can be accessed using menu items, the number of options for each of them is different. While the *Grid* feature simply can be turned ‘on’

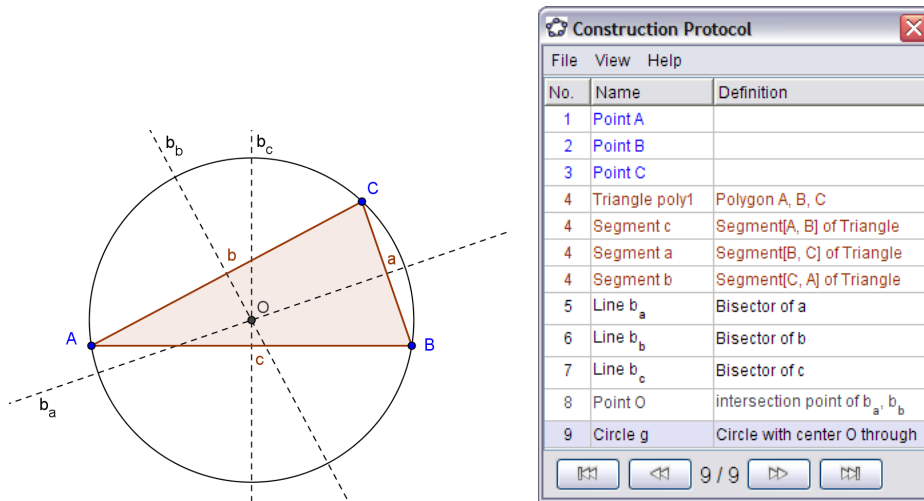


Figure 7.3: Sketch and construction protocol

and ‘off’, there are four different options available for the *Point capturing* feature. This could be a reason for the different difficulty rating of the two features and needs to be addressed during future introductory workshops. Features like *Point capturing* need to be introduced more carefully by explaining and trying out its different setting options to help participants understand the way they work.

7.2.2 Context Menu Features

The *Context menu* provides a list of features that can be applied to the selected object, for example showing or hiding objects or their labels, turning the trace on or off, renaming or redefining objects (see figure 7.4).

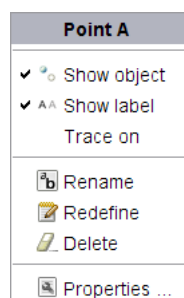


Figure 7.4: Context menu

On average, the difficulty level of accessing GeoGebra’s *Context menu* was rated by the participants to be $\bar{x} = 1.38$ with a standard deviation of $s = 1.39$. The *Context menu* can be opened by right clicking an object, which is possible for a touchpad or mouse with

two keys (e.g. MS Windows computers). Alternately, it can be accessed by holding the *Ctrl*-key pressed while clicking an object, which is necessary for a touchpad or mouse with just one key (e.g. MacOS computers).

When analyzing the two different ways of accessing the *Context menu* the following observation was made: In general mouse users rated this feature with $\bar{x} = 0.71$ ($s = 0.95$) considerably easier than touchpad users with an average rating of $\bar{x} = 1.61$ ($s = 1.43$). Unfortunately it wasn't possible to find out how many of the mouse users actually were equipped with a one-key mouse because it can be easily replaced by a two-key mouse and the other way round. Instead, the difficulty ratings of the *Context menu* for one-key and two-keys touchpad users were analyzed, who together represented 82.1% of the workshop participants. Considering the type of operating system installed on the teachers' computers, it was possible to determine that 53.1% of the touchpad users had to use the *Ctrl*-click method while 46.9% could use the right-click method in order to access the *Context menu* (see table 7.2). With an average rating of $\bar{x} = 1.31$ and a standard deviation of $s = 1.20$, the first group (touchpad with one key, *Ctrl*-click) rated accessing the *Context menu* considerably easier than the second group (touchpad with two keys, right click) with a mean of $\bar{x} = 1.93$ and a standard deviation of $s = 1.62$.

Input Device	Participants	Access	Percent	Mean	StDev
Mouse	17.9%	–	–	0.71	0.95
Touchpad	82.1%	<i>Ctrl</i> -click	53.1%	1.31	1.20
		right click	46.9%	1.93	1.62

Table 7.2: Access to *Context menu* rated by mouse and touchpad users

Although the rating difference between the two groups is not practically significant due to the wide distribution of the data, it is obvious, that the participants found it easier to access the *Context menu* by using a *Ctrl*-click than a right click. A possible reason for this result is that it's probably easier for them to distinguish this special key-click-combination from a 'usual' (left) click. By contrast, users of touchpads with two keys need to press the right key in order to open the *Context menu* which is very similar to a 'usual' (left) click. Therefore, both clicks can easily be mixed up by unexperienced users which probably causes additional difficulties when opening the *Context menu*.

Trace On

The *Trace on* feature was rated within the 'middle' features group with an average rating of $\bar{x} = 1.29$ and a standard deviation of $s = 1.21$. After opening the *Context menu* for a certain object, the *Trace on* feature can be selected. This causes the object to leave its trace whenever it is moved either directly by dragging it with the mouse, or indirectly by modifying one of its parent objects. The object's trace isn't displayed in the algebra window and even in the graphics window the trace is a temporary phenomenon. This means that whenever the view of the graphics window is refreshed, the trace disappears. Furthermore,

the trace of an object can't be saved with a construction. These characteristics of the *Trace on* feature caused a lot of confusion among the workshop participants, and therefore, they need to be addressed more clearly during future introductory workshops.

Rename and Redefine

Feature *Rename* was introduced during the first workshop and rated to be part of the 'middle' features group with an average rating of $\bar{x} = 1.30$ and a standard deviation of $s = 1.42$. After selecting this feature in the *Context menu* for a certain object, a dialog window appears allowing users to type in the new name of the object.

Feature *Redefine* was introduced during the third workshop and rated to be one of the most 'difficult' features with an average rating of $\bar{x} = 1.77$ and a standard deviation of $s = 1.49$. This feature also can be accessed over the *Context menu*. A dialog window is opened that contains the definition of the selected object which can be modified by the user.

Both features *Rename* and *Redefine* can be selected from the *Context menu* which opens a dialog window in both cases. At first glance, this makes them very similar to use and doesn't support the different difficulty ratings for these features. Nevertheless, there are some differences between the two features that need to be mentioned at this point. In order to rename an object, just the new name needs to be entered into the input field of the corresponding dialog window and confirmed by clicking 'Apply'. Additionally, there is a 'fast renaming' option which avoids opening the *Context menu* at all: After selecting an object in *Move* mode, starting to type the new name opens the corresponding dialog window.

By contrast, the *Redefine* feature requires more knowledge about available commands as well as the names and relations of objects already used within the construction. Redefining an object not only affects the object itself but can also have impact on dependent objects. Although it is a very useful tool for rearranging a construction that already consists of several objects, the *Redefine* feature has to be applied very carefully and is considered to be an advanced feature whose introduction needs to be well prepared.

Since these two features seem to be quite similar from the point of view of a GeoGebra beginner, they often get mixed up. Experience with previous GeoGebra introductory workshops suggests that participants often have difficulties distinguishing these two features, not least because they also have similar names. Therefore, the *Rename* feature should be introduced in the beginning using the 'fast renaming' option. Without having to open the *Context menu*, participants won't get confused about the two features and don't choose the wrong one by accident. Whenever the participants seem to be ready to learn about the *Redefine* feature, it makes sense to point out the similarities and differences between the two features. By all means it is important to help the teachers understand the different concepts behind these features in order to make it easier for them to apply them correctly.

7.2.3 Properties Dialog Features

GeoGebra's *Properties dialog* can be accessed in three different ways, either by selecting it from the *Edit* menu, or from the *Context menu*, or by double clicking on an object in *Move* mode. Once the *Properties dialog* window is opened, users can change the properties of all existing objects (see figure 7.5). The *Properties dialog* was rated to be part of the 'easy' features group with an average rating of $\bar{x} = 1.20$ and a standard deviation of $s = 1.22$.

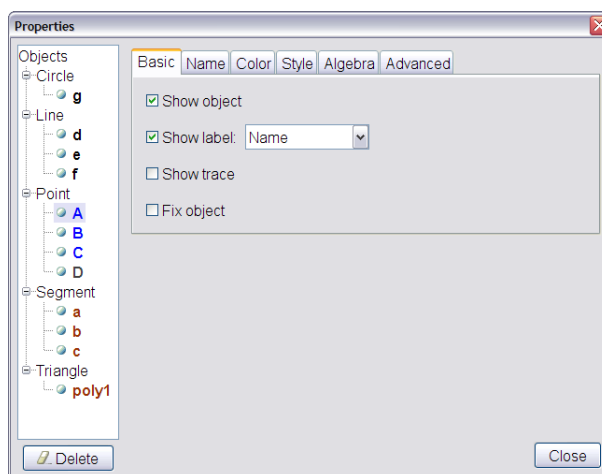


Figure 7.5: Properties dialog

Figure 7.5 shows a usual *Properties dialog* for a GeoGebra construction. On the left hand side, a list of all existing objects is displayed grouped by object types. Depending on which of the objects is highlighted, a selection of tabs is shown on the right hand side that allow the user to modify the properties of the selected object.

Label Types

Changing an object's label was rated to be part of the 'middle' features group with an average difficulty rating of $\bar{x} = 1.35$ and a standard deviation of $s = 1.25$. After opening the *Properties dialog* and selecting a certain object in the list, the visibility and type of label for this object can be changed.

A possible explanation for this difficulty rating is that in addition to showing or hiding the label of an object, there are different options for the type of label. Thus, users can choose if they want to display the *Name*, *Name & value*, or just *Value* of the object within the graphics window. Since this can confuse beginners, they should get the chance to try out these three different label types for several types of objects. The meaning of 'value' for different types of objects seems to cause additional troubles, and thus, the distinct interpretations should be given some special attention during future introductory workshops.

Background Image

The *Background image* feature was introduced in the second workshop and rated to be part of the ‘difficult’ features group with an average rating of $\bar{x} = 1.63$ and a standard deviation of $s = 1.32$. This feature can be applied to an already inserted picture by checking the option *Background image* on tab *Basic* in the *Properties dialog*.

Although setting a background image shouldn’t be much of a challenge, its difficulty rating definitely makes sense especially in view of the fact that it can only be applied to an already inserted image. Many participants had difficulties finding a suitable picture on the Internet and inserting it into GeoGebra (the *Insert image* tool was rated as one of the two most ‘difficult to use’ tools). These difficulties prevented many participants from inserting the required image and therefore, they were unable to try out feature *Background image* themselves. In future workshops, all necessary files used (e.g. pictures) should be provided on a webpage, CD, or USB drive in order to prevent such circumstances that cause additional and unnecessary difficulties for the participants.

Auxiliary Objects

The *Auxiliary objects* feature was introduced during workshop III and rated by the participants to be the most difficult of the introduced GeoGebra features with an average rating of $\bar{x} = 2.05$ and a standard deviation of $s = 1.51$. Each object can be defined as an *auxiliary object* by checking the corresponding item on the *Basic* tab of its *Properties dialog*. By this means, it is removed from the group of free or dependant objects and placed in a third group called *Auxiliary objects*. Using the *View* menu the whole group can either be displayed in or hidden from the algebra window.

Purpose of this feature is to keep the algebra window free of objects that would confuse students when working with a GeoGebra construction provided by the teacher. During the introductory workshop, feature *Auxiliary object* was introduced by the presenter along the way in order to ‘clean up’ the algebra window during the construction of a slope triangle. Although the participants could watch the presenter applying this feature, they didn’t get the chance to try it out themselves, which could be a reason for the difficulty rating of this feature.

7.3 Impact of External Variables

In this section, the potential impact of external variables on the difficulty ratings of workshop activities, home exercises, tools, commands, as well as features of GeoGebra is examined. After grouping the participants according to certain characteristics, such as age, mathematics content knowledge, or computer skills, potential differences are analyzed between the difficulty ratings of these groups. This analysis will help to determine if GeoGebra introductory workshops are equally suitable for different groups of participants and if learning how to use GeoGebra is affected by external variables.

7.3.1 Gender and Age

The group of 44 workshop participants was composed of 35 female (79.5%) and 9 male (20.5%) secondary school teachers. The average difficulty ratings of women and men are very similar for all rated workshop tasks and GeoGebra components: workshop activities (1), home exercises (2), GeoGebra tools on day of introduction (3), commands on day of introduction (4), and GeoGebra features (5) (see figure 7.6(a)). Additionally, a statistical comparison of the average ratings of male and female workshop participants didn't reveal any significant differences.

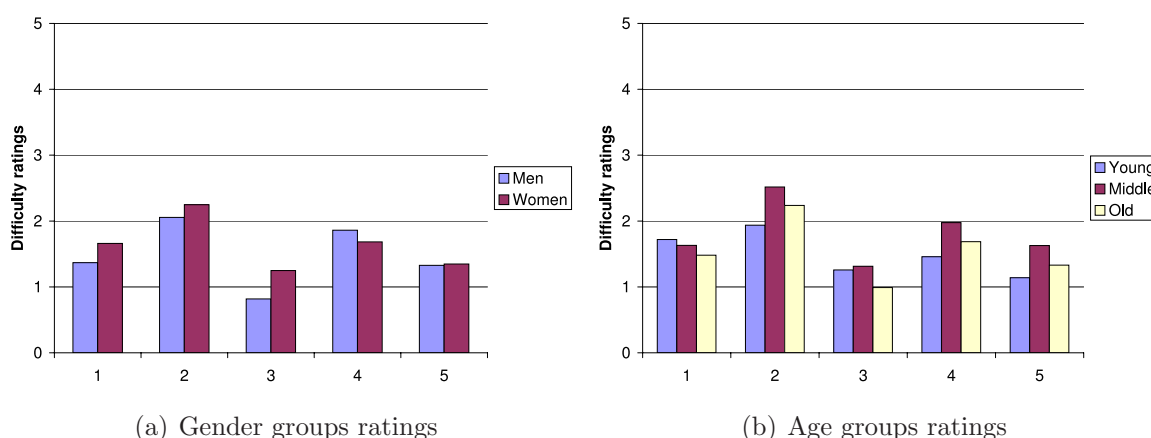


Figure 7.6: Average difficulty ratings for workshop tasks and GeoGebra components

In order to check the difficulty ratings for different age groups, thresholds for ‘young’, ‘middle’, and ‘old’ participants had to be defined. Therefore, the average age of 38.62 years plus / minus half of the standard deviation ($s = 10.0$) was used resulting in about an even distribution of participants for all three age groups (see table 7.3). Again, the average ratings of the three age groups for the workshop tasks and GeoGebra components mentioned above were compared (see figure 7.6(b)) and no noteworthy differences between the age groups could be found. This was confirmed by a statistical comparison using a Kruskal Wallis test for three independent samples.

Group	Age range	Members		Computer Skills	
		Number	%	Score	%
‘Young’ participants	24.0 – 33.5	14	33.3%	32.3	76.9%
‘Middle’ participants	33.6 – 43.6	15	35.7%	28.6	68.1%
‘Old’ participants	43.7 – 59.0	13	31.0%	27.9	66.4%

Table 7.3: Age groups of participants

Considering these similar difficulty ratings for all groups examined so far, the conclusion is drawn that neither gender, nor age have any impact on the subjective rating of GeoGebra

and its components, as well as the introductory workshops in general. This is especially interesting for the different age groups of participants, since ‘younger’ participants were estimated to be more familiar with using a computer and therefore, to have higher computer skills and less difficulties using GeoGebra than ‘older’ participants. This estimation was supported by the average computer skills score of the ‘young’ participants group which was higher than the score of the ‘old’ participants group (see table 7.3). Also, a statistical comparison of these average computer skills scores confirmed this observation about the computer skills of ‘young’ and ‘old’ workshop participants (one-tailed Mann-Whitney test, $\alpha = 0.031$, $U = 52.500$, $Z = -1.889$). Thus, the computer skills of participants, also seem not to have any impact on the difficulty ratings of the workshops and GeoGebra itself (see section 7.3.4).

7.3.2 Teaching Experience

Out of 41 workshop participants who specified which grade levels they were teaching, 30 were middle school (73.2%) and 11 were high school teachers (26.8%). Figure 7.7(a) shows the average difficulty ratings for middle and high school teachers for the workshop tasks and GeoGebra components mentioned before¹. Again, no differences between middle and high school teachers could be found, indicating that the difficulty level of using GeoGebra is appropriate for all secondary school mathematics teachers.

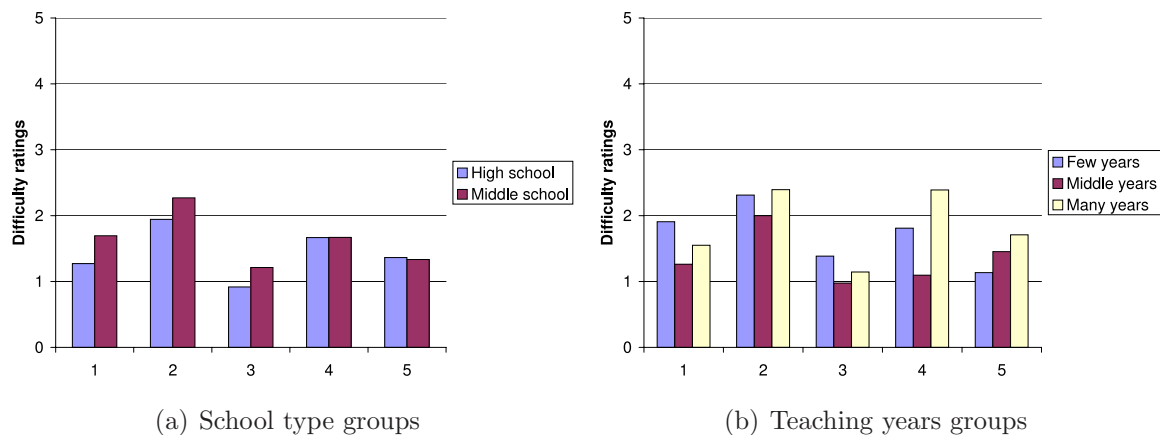


Figure 7.7: Average difficulty ratings for workshop tasks and GeoGebra components

As another measure for the teaching experience of the participating teachers, they were assigned to three groups according to their number of years as a teacher. The thresholds for ‘few’ years and ‘many’ years groups were defined as the mean of all teachers ($\bar{x} = 7.68$ years) plus / minus half of the standard deviation ($s = 6.79$), which created the distribution of participants shown in table 7.4.

¹Workshop activities (1), Home exercises (2), GeoGebra tools on day of introduction (3), Commands on day of introduction (4), and GeoGebra features (5)

Group	Years range	Members	Percentage
'Few' years	1 – 4	18	42.8%
'Middle' years	5 – 11	13	31.0%
'Many' years	12 – 29	11	26.2%

Table 7.4: Teaching years groups of participants

A comparison of the average difficulty ratings for workshop tasks and GeoGebra components revealed similar difficulty ratings for all groups (see figure 7.7(b)), which was confirmed by a statistical comparison using a Kruskal Wallis test for three independent samples. Therefore, teaching experience doesn't seem to have any impact on the subjective difficulty rating of the introductory workshops and GeoGebra itself.

7.3.3 Mathematics Content Knowledge

Out of 44 participating secondary school teachers, 38 filled in the mathematics content knowledge survey (survey 3, see section 4.4.2). The high score for this survey was 25 points², which was reached by one participant. The lowest score was 6 points, while on average participants reached a score of 16.7 points.

Concerning their mathematics content knowledge, the participants were assigned to three groups according to their individual score. The average score of all participants $\bar{x} = 16.7$ plus / minus half of the standard deviation ($s = 4.7$) was chosen in order to get thresholds for the 'low' and 'high' content knowledge groups (see table 7.5). In this way, 12 teachers were assigned to the 'low' content knowledge group and 13 participants formed the 'middle' group, while the 'high' content knowledge group also consisted of 13 members.

Group	Score	Members	Percentage
Low math content knowledge	6 – 14	12	31.6%
Middle math content knowledge	15 – 19	13	34.2%
High math content knowledge	20 – 25	13	34.2%

Table 7.5: Mathematics content knowledge groups

A closer look at the average difficulty ratings for the workshop tasks and GeoGebra components mentioned before³ indicated that the average ratings were quite similar for all groups (see figure 7.8). This was confirmed by a Kruskal Wallis test for three independent samples which didn't reveal any noteworthy differences between the mathematics content knowledge groups. Thus, the mathematics content knowledge of teachers seems not to have

²Score corresponds to number of correctly solved problems

³Workshop activities (1), Home exercises (2), GeoGebra tools on day of introduction (3), Commands on day of introduction (4), and GeoGebra features (5)

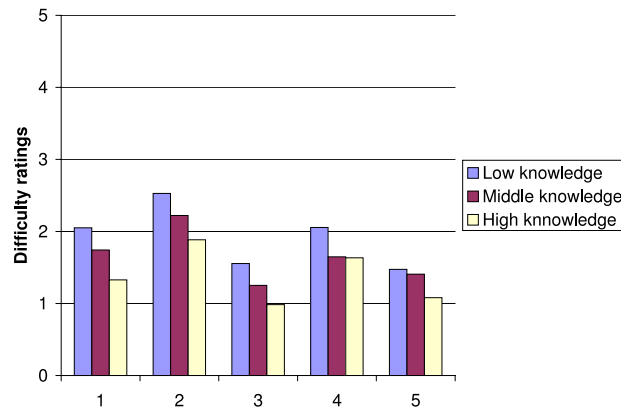


Figure 7.8: Content knowledge and average difficulty ratings

any impact on the subjective ratings of the introductory workshops as well as GeoGebra itself.

7.3.4 General Computer Skills

In order to find a measure for the computer skills of the participants, a list of computer activities that consisted of 17 items was created. In survey I, participants had to check whether they knew how to carry out the corresponding activity or not in order to produce a very subjective estimation of their computer skills.

Table 7.6 shows the list of computer activity items as well as the percentage of participants who stated to know how to carry out the corresponding activity. Since the computer skills activities ranged from a simple left click with the mouse to using html code in order to create web pages, the different items were weighted in order to satisfy their varying difficulty levels. Thus, a high score of 42 points could be reached, which produced a measure for the subjective estimation of computer skills for every participant.

Concerning their computer skills, the participants of the introductory workshops scored within a range from 11 points to 42 points. According to the individual scores, the participants were assigned to three groups with ‘low’, ‘middle’, and ‘high’ computer skills. The average score of $\bar{x} = 29.8$ points plus / minus half of the standard deviation ($s = 7.4$) was chosen in order to get thresholds for the ‘low’ and ‘high’ computer skills groups. Thus, the distribution of participants within the three groups listed in table 7.7 was created.

Figure 7.9 shows the average difficulty ratings for the workshop tasks and GeoGebra components mentioned before⁴, which again, seem to be very similar for all three groups. This was confirmed by a Kruskal Wallis test for three independent samples which didn’t reveal any noteworthy differences between the computer skills groups. Therefore, it seems

⁴Workshop activities (1), Home exercises (2), GeoGebra tools on day of introduction (3), Commands on day of introduction (4), and GeoGebra features (5)

Nr.	I know how to...	Percentage	Weight
1	Left click (MacOS: click)	81.4%	1
2	Right click (MacOS: Apple-click)	81.0%	1
3	Click and drag	97.7%	1
4	Save a file	100%	2
5	Delete a file	97.7%	2
6	Copy and paste text	95.3%	2
7	Print a file	86.0%	2
8	Create a folder	90.7%	2
9	Select a file or folder	97.7%	2
10	Open a web browser	93.9%	2
11	Select a series of files or folders	90.7%	3
12	Make a screenshot	37.2%	3
13	Insert pictures into an MS Word file	79.1%	3
14	Find information on the Internet	100%	3
15	Upload files to the Internet	69.8%	4
16	Create a web page	27.9%	4
17	Use html code	16.3%	5

Table 7.6: Computer skills items

Group	Score	Members	Percentage
'Low' computer skills	11 – 26	14	33.3%
'Middle' computer skills	27 – 33	17	40.5%
'High' computer skills	34 – 42	11	26.2%

Table 7.7: Computer skills groups

that the subjective estimated computer skills don't have any impact on the difficulty ratings of the introductory workshops as well as GeoGebra itself, which definitely argues for the user friendliness of the software as well as a user-oriented design of the introductory workshops.

7.3.5 Mouse vs. Touchpad Use

In the Broward County School District, many public schools are equipped with Apple notebook computers, which can be used by students as well as teachers. Additionally, most of the teachers were given a MacOS notebook computer from their school to support their administrative work and classroom teaching. Therefore, more than half of the participants (56.1%) were using a notebook computer with the operating system MacOS, while 43.9% of the teachers used MS Windows notebook computers.

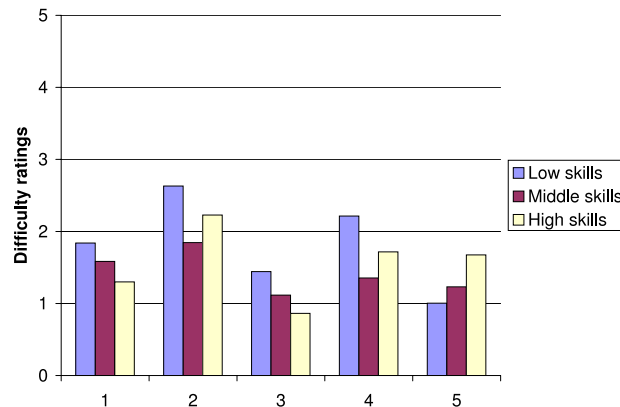


Figure 7.9: Computer skills and average difficulty ratings

Again, the average difficulty ratings for workshop tasks and GeoGebra components⁵ were examined and displayed in figure 7.10(a). Comparing the average ratings of both groups, no differences between users of MacOS and MS Windows computers were found, which was supported by a Mann-Whitney test. The conclusion was drawn, that both versions of GeoGebra (for MS Windows and MacOS operating system) are similarly easy to use and that there is no need to distinguish more clearly between these two user groups during introductory workshops⁶.

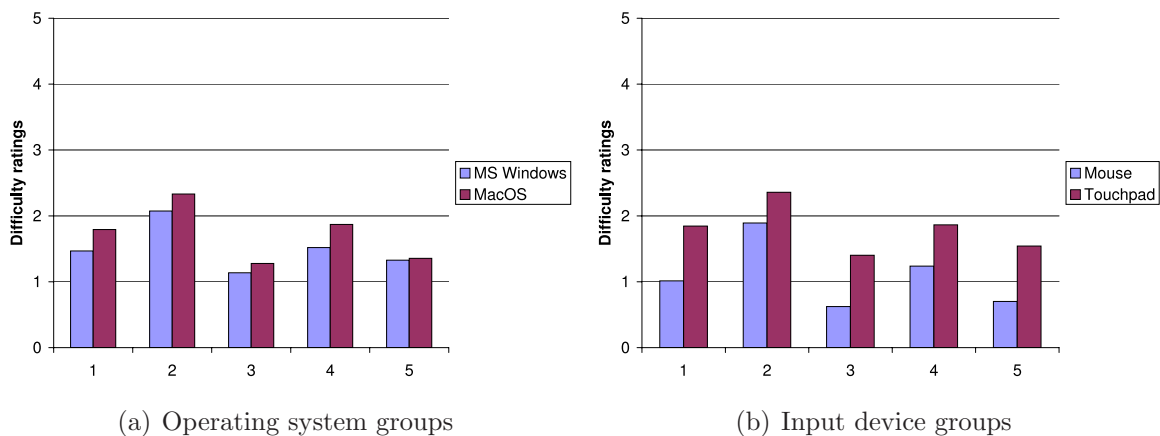


Figure 7.10: Average difficulty ratings for workshop tasks and GeoGebra components

⁵Workshop activities (1), Home exercises (2), GeoGebra tools on day of introduction (3), Commands on day of introduction (4), and GeoGebra features (5)

⁶If any obvious differences concerning the operation of GeoGebra for MacOS occurred, this was addressed during the workshop right away (e.g. accessing the *Context menu*). Additionally, useful hints and tricks for both, MS Windows and MacOS users, were given during the workshops.

Finally, also the input device used to operate GeoGebra during the introductory workshops was given a closer look. 17.9% of the workshop participants used a mouse while 82.1% of the teachers used the touchpad of their notebook computers when creating constructions with the dynamic geometry tools. Figure 7.10(b) displays the average difficulty ratings for these two groups of participants for the workshop tasks and GeoGebra components mentioned before.

Contrary to all the other external variables analyzed before, there are considerable differences between some of the difficulty ratings of mouse and touchpad users. Figure 7.11 shows the average ratings for mouse and touchpad users together with the standard deviations for each data point. For items 1 (workshop activities), 3 (tools), and 5 (GeoGebra features) the standard deviations don't overlap which indicates a real significant difference between these average ratings.

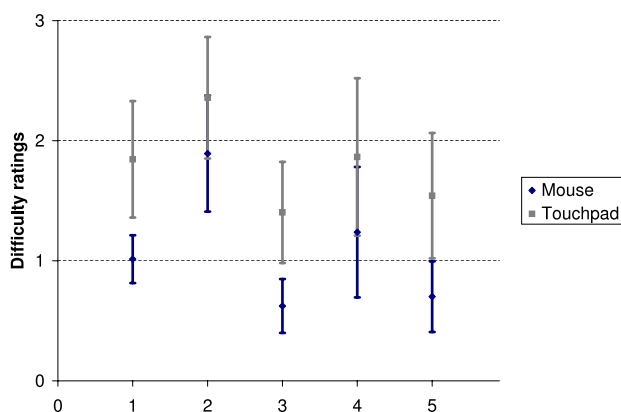


Figure 7.11: Input device groups difficulty rating differences

Workshop activities

The first difference pertains to the workshop activities ratings. While the group of mouse users rated the workshop activities with $\bar{x} = 1.01$ and a standard deviation of $s = 0.40$, the group of touchpad users found them more challenging with an average rating of $\bar{x} = 1.84$ and a standard deviation of $s = 0.97$. When these two average difficulty ratings were compared statistically, a one-tailed Mann-Whitney test revealed a significant difference on an $\alpha = 0.012$ level ($U = 51.50$, $Z = -2.215$).

This means that, in general, participants who used a touchpad during the series of introductory workshops found the workshop activities more challenging than teachers who used a mouse. This indicates that the touchpad causes additional problems for the participants and therefore makes the workshop activities seem to be more difficult than they really are. Concerning the home exercises, no significant difference could be found. Although this couldn't be confirmed by the data, it is likely that many teachers used their desktop

computers instead of their notebook computers at home in order to do the home exercise. Since a desktop computer is usually operated with a mouse, the additional difficulties didn't occur and therefore didn't influence the home exercise ratings.

Dynamic Geometry Tools

Concerning the dynamic geometry tools introduced during the workshop series, another significant difference between mouse and touchpad users could be found. Mouse users rated the tools on the day of their introduction on average with $\bar{x} = 0.62$ ($s = 0.45$) while touchpad users found them more challenging with a mean of $\bar{x} = 1.40$ ($s = 0.84$). When these average difficulty ratings were compared statistically, a Mann-Whitney test revealed a significant difference on an $\alpha = 0.011$ level (one-tailed, $U = 50.0$, $Z = -2.270$).

This means on average that touchpad users had more trouble operating the newly introduced GeoGebra tools than mouse users which made the tools seem to be more difficult to use than necessary. Experience indicates that usually it is far trickier to operate GeoGebra with a touchpad than with a mouse even for experienced GeoGebra users, and especially if someone is not used to working with a touchpad. Operating this input device requires fine motor skills that are very different from the ones necessary to working with a mouse and therefore need a lot more practice.

For example, dragging an object using a touchpad, which is necessary all the time when using dynamic geometry software, is rather tricky. While the key needs to be pressed and held, one has to use another finger in order to actually move the object to another position. Although participants were rather inventive with finding solutions for this problem (the use-the-other-hand-too-solution was very popular but required coordination of both hands), just watching them trying again and again required a lot of patience.

GeoGebra Features

The last significant difference was found between the difficulty ratings of GeoGebra features for the two input device groups. While mouse users rated the eleven GeoGebra features with $\bar{x} = 0.70$ on average ($s = 0.59$), the group of touchpad users rated them to be more difficult with an average of $\bar{x} = 1.54$ and a standard deviation of $s = 1.04$. A statistical comparison of these two difficulty ratings revealed a significant difference on an $\alpha = 0.019$ level in a one-tailed Mann-Whitney test ($U = 55.50$, $Z = -2.069$).

Concerning the GeoGebra features, the touchpad turned out to influence the experienced difficulty level and to make it more challenging for participants to work with GeoGebra. Since applying the features to existing objects requires precise clicks (e.g. selecting the object, checking items in the *Properties dialog*) the touchpad seems to be rather inefficient and causes a lot of frustration among new GeoGebra users.

As a solution for these problems computer mice should be available for the participants during future introductory workshops and touchpad issues should be discussed right in the beginning of the workshop. If participants know about the additional difficulties they are

about to experience when using the touchpad, they are more likely to switch to using a mouse and won't blame the software for their inability to properly carry out activities.

7.4 Observations during Introductory Workshops

General Review of Helper Reports

During the series of four introductory workshops for GeoGebra, several helpers were assigned to each of the three groups of participants in order to assist the presenter and to support the participants if necessary. They filled in so called 'Helper report cards' characterizing and explaining the problems to which they responded. Although not all of the helpers were reliable with reporting the difficulties and needed to be asked frequently to fill in the cards, a total of 309 problems were reported during the four workshop days. That amounted to an average of 77.25 reports per day and 25.75 reports per workshop group.

Table 7.8 lists the number of reports per workshop, as well as their characterization as a mathematical, GeoGebra related, or operating system related difficulty, and the average number of such interventions per workshop day. The same difficulty could refer to one or more categories, for example a GeoGebra problem that just occurred on a MacOS notebook computer. On average, 68.25 reports per workshop were related to GeoGebra questions, while 36.75 reports per workshop pertained to the differences between MS Windows and MacOS notebook computers and / or the corresponding GeoGebra versions. Only 17 reports per workshop were characterized as mathematical content problems, which supports the appropriateness of the workshop contents for secondary school teachers.

On day four the number of difficulties reported decreased by about 50% compared to the other days. Unfortunately this doesn't necessarily imply that there were less problems in workshop 4 than in previous workshops since the reduction may have been caused by the definitely decreasing motivation of helpers to write down participant difficulties.

Workshop	Reports	Math	GeoGebra	MS Windows	MacOS
WS I	95	12	76	29	17
WS II	79	2	72	14	14
WS III	91	1	89	29	18
WS IV	44	2	36	16	10
Mean / WS	77.25	4.25	68.25	22	14.75

Table 7.8: Characterization of difficulties reported by helpers

Detailed Review of Helper Reports

Using a Grounded Theory approach [Strauss and Corbin, 1998], helper feedback was systematically analyzed by organizing their notes into 18 keywords grouped into six cate-

gories: ‘Mathematical Content’, ‘Computer Issues’, ‘GeoGebra in General’, ‘GeoGebra Tools’, ‘GeoGebra Features’, and ‘Algebraic Input and Commands’.

Helper Feedback		
<i>Category</i>	<i>Keyword</i>	<i>Frequency</i>
(a) Mathematical Content	1 Activities	32
(b) Computer Issues	2 Installation	9
	3 Files	9
	4 Input device	4
	5 MS Word	7
(c) GeoGebra in General	6 Basic handling	6
	7 GeoGebra concepts	26
	8 Selecting objects	13
	9 Other issues	13
(d) GeoGebra Tools	10 Toolbar	9
	11 Use of tools	35
	12 Challenging tools	71
(e) GeoGebra Features	13 Menu bar	16
	14 Properties dialog	20
	15 Context menu	17
	16 Other features	11
(f) Algebraic Input and Commands	17 Input syntax	28
	18 Use of commands	9

Table 7.9: Coding and frequencies for helper feedback

Mathematical Content: In this category feedback about the mathematical content of the workshop activities is summarized.

- *Activities:* 32 helper reports were related to the workshop activities. 28 dealt with helping the participants to understand the construction process which was unfamiliar to some of them. Four reports pertained to the paper-and-pencil task of the first workshop. They stated that participants had troubles operating the compass and straightedge in order to create a line bisector.

Since the mathematical content knowledge of workshop participants can vary over a wide range, dynamic constructions need to be introduced more carefully. The presenter should summarize geometric properties of the corresponding figure as well as the required construction steps with the participants before actually beginning with the construction. Once the construction process was discussed, the participants know the course of actions and should be better prepared to actually create the dynamic figure.

Computer issues: In this category general computer issues that occurred during the introductory workshops are summarized.

- *Installation:* Nine reports pertained to the installation of GeoGebra. Due to problems with the Internet connection of the notebook computers, the installation files for GeoGebra were distributed via USB-drives. Since the installation files and required actions were different for MS Windows and MacOS notebooks, some participants became confused and had issues installing GeoGebra on their notebook computers.

Usually, the installation of GeoGebra is no problem if all computers used during the workshops have an Internet connection. This allows the participants to use GeoGebra WebStart in order to install the software on the computers. Since no special user permissions are required for this installation option, this usually works in computer labs. If no Internet connection can be guaranteed, helpers need to be available at the beginning of a workshop who are proficient in handling files on different operating systems.

- *Files:* 9 reports dealt with difficulties related to working with files and folders. Since many teachers used their school notebook computers, their permissions to save files on the hard disk were restricted. Therefore, they had troubles saving their GeoGebra files and needed help to create folders for them. Additionally, some teachers couldn't remember where they saved their files and therefore, couldn't find them later on.

Since these issues are not connected with GeoGebra in particular, they usually can't be prevented during a workshop. Nevertheless, creating a GeoGebra folder on the desktop in the beginning of a workshop could help the participants to avoid these difficulties.

- *Input device:* 4 reports were related to the input device participants used to operate GeoGebra. Helpers had to explain the proper use of a touchpad and assist with operating the mouse. On the one hand, participants were not used to operating a touchpad or mouse in such a detailed environment like a GeoGebra construction and therefore, they lacked the necessary motor skills. On the other hand, some participants with MS Windows notebook computers had difficulties distinguishing between left and right clicks, which caused additional problems. As mentioned before, the different difficulty ratings of workshop participants using a touchpad compared to the ones using a mouse in order to operate GeoGebra indicate that a touchpad causes more trouble than a mouse. Therefore, the workshop presenter should provide several computer mice for the participants in order to prevent additional difficulties and facilitate the first contact with GeoGebra.
- *MS Word:* 7 reports dealt with issues concerning the general use of MS Word as well as inserting pictures into a text processing document. Since participants have different levels of previous knowledge about using text processing software,

a detailed handout explaining how to insert pictures, create tables, and insert formulas into MS Word could be helpful and reduce the number of helpers needed for support.

GeoGebra in General: In this category general GeoGebra issues and characteristics that caused problems among participants are summarized.

- *Basic handling:* 6 helper reports were related to starting GeoGebra itself as well as opening GeoGebra files. Some participants kept clicking on the installer file, causing the computer to re-install GeoGebra every time they wanted to start it. Other participants had troubles opening prepared GeoGebra constructions or a new GeoGebra window.

The difficulties concerning starting GeoGebra were related to the missing Internet connection. Since the software was distributed using installer files, some participants saved them on their desktop and mixed them up with the actual GeoGebra icon of the installed software. If installing issues occur during a workshop, this needs to be addressed by the presenter asking the participants to relocate the installer file and explaining the differences between opening this file and the actual software.

- *GeoGebra concepts:* 26 helper reports dealt with general GeoGebra concepts that confused participants. A lot of teachers kept creating objects accidentally because they forgot to go back to *Move* mode before actually moving an object and therefore asked the helpers for support. Additionally, the difference between free and dependant objects in GeoGebra caused a lot of confusion. Since participants didn't understand why GeoGebra makes this distinction, they tried to move dependant objects and wondered why this wouldn't work. They usually blamed the software for this 'mistake'.

In order to prevent the confusion about accidentally creating new objects, presenters should address this issue frequently during future introductory workshops and remind participants of checking the toolbar help in order to find out which tool is currently activated before trying to move an object.

In order to prevent the issues related to trying to move dependant objects with the mouse, a modification of GeoGebra's behavior could be considered: a pop-up text message could be implemented which indicates that the intended action can't be performed because the selected object depends on other objects.

- *Selecting objects:* In 13 reports helpers described issues related to the selection of one or several objects in GeoGebra. On the one hand, participants kept creating points because they didn't wait for an already existing one to highlight when moving the mouse over it. On the other hand, participants had troubles with objects that lay on top of each other. This was especially challenging for teachers if they tried to select such an object, which caused GeoGebra to open a window containing a list of all objects in this position asking the user to select one of them.

Again, these topics need to be addressed more clearly by the presenter in order to prevent these issues from happening. The selection window especially requires some explanations because participants seem to be afraid of pop-up windows and tend to close them without reading the message inside. Unfortunately, closing the window prevents them from actually finishing the task they wanted to carry out.

- *Other issues*: 13 reports pertained to other issues that were connected with the general use of GeoGebra. Helpers had to explain methods of deleting objects in GeoGebra and give hints concerning the use of keyboard shortcuts that facilitate using the software. For example “The *Esc*-key activates *Move* mode”. Additionally, participants wanted to know how to save settings as well as how to increase the font size for presentation purposes.

GeoGebra Tools: In this category feedback about GeoGebra tools is summarized.

- *Toolbar*: Nine helpers reported that they had to assist with the proper use of the toolbar. On the one hand, participants kept opening the toolboxes although the icon of the corresponding button already showed the tool they wanted to use and a single click would have been sufficient in order to activate it. On the other hand, teachers mixed up certain tools (e.g. *Line through two points* and *Ray through two points*), whose icons look similar to each other.

In order to prevent users from searching the toolbar for a certain tool, a handout listing all the toolboxes as well as their corresponding tools and icons could help participants to get a better overview about the organization of the toolbar. The relation between tools of each toolbar should be explained by the presenter. Additionally, a digital version of this handout could be included into the *Help* menu of GeoGebra.

- *Use of Tools*: 35 reports pertained to the proper use of tools which were already introduced to the participants. Although the presenter asked them to read the toolbar help after selecting a tool in order to find out how to operate it, participants seemed to prefer asking one of the helpers to explain the use of an unfamiliar tool. Therefore, more emphasis needs to be placed on explaining the importance of the toolbar help to the participants in order to prepare them for using GeoGebra on their own without having someone around to ask about the proper use of tools.
- *Challenging tools*: 71 helper reports were related to certain tools that seemed to be challenging for the participants. Helpers often had to explain the two ways of applying the *Intersect two objects* tool to the participants, as well as how to properly use the general tools *Zoom* and *Move drawing pad*. Additionally, the difference between moving a slider and changing its value by moving the point on the slider, as well as showing and hiding objects and their labels by using the corresponding tools required considerable additional support.

Furthermore, participants seemed to have a lot of trouble understanding the difference between a clockwise and counterclockwise orientation of a polygon, which causes GeoGebra to either display its ‘exterior’ or ‘interior’ angles for each vertex. This behavior is related to the functionality of the *Angle* tool which always creates an angle in mathematically positive direction. The resulting angle depends on the order of selecting or creating the required points, and by default reflex angles are allowed as well. Although this setting can be changed in the *Properties dialog* for already existing angles, it is currently not possible to change this default setting for angles in the *Options* menu. This modification of GeoGebra might be implemented in a future version in order to prevent difficulties related to the orientation of angles for teachers as well as students.

Although the number of helpers available for support probably might have encouraged the participants to ask for their help all the time, the presenter should set up some kind of rules concerning the appropriate behavior during a workshop. Since usually the presenters are on their own or supported by one assistant, participants might be instructed to address their neighbor first if difficulties occur. Chances are that a colleague already knows the answer to the problem and therefore could solve the issue without having to ask the presenter / assistant. Nevertheless, if assistants have to deal with problems of general interest, they should be allowed to interrupt the presentation and explain the solution to the whole group in order to avoid having to answer the same questions several times.

GeoGebra Features: In this category feedback about GeoGebra features that were challenging for the participants and therefore required a lot of support from the helpers are summarized

- *Menubar*: 16 helper reports pertained to GeoGebra features that can be accessed over the menubar. Participants wanted to know how to show or hide the *algebra window* as well as the *grid* and needed help when changing the *Point capturing* options. Additionally, support was needed in order to show or hide the coordinate axes as well as to change their scale.

Although all this information was already given by the instructor, participants kept asking the helpers about it. Again, a detailed handout summarizing how to deal with the features from the menubar could help the participants and reduce the number of questions that need to be answered by helpers during a workshop.

- *Properties dialog*: 20 reports were related to the *Properties dialog* of GeoGebra. Teachers needed support when changing the properties of certain objects (e.g. set a background image, show the value of an object) and wanted to know how to select several objects or all objects of the same type within the list of objects provided in the *Properties dialog* window. Additionally, participants became confused when they tried to open the *Properties dialog* for an image

that was already set as a background image in order to change its filling. Since a background image no longer responds to right-clicking, the *Properties dialog* needs to be opened either from the *Edit* menu, or by right-clicking another object and selecting the picture afterwards in the list of objects.

Furthermore, the difference between the ‘usual’ *Properties dialog* and the one for the drawing pad was not clear for some of the teachers and therefore, caused additional trouble. Participants didn’t right-click directly on an object, but on the drawing pad which caused the ‘wrong’ *Properties dialog* to open and confused the teachers since they couldn’t find the list of objects from which they wanted to choose.

- *Context menu*: In 17 reports helpers stated that participants needed support when dealing with features that can be accessed from the *Context menu*. Apart from having trouble renaming and redefining objects, most of the issues were connected to the *Trace* feature of GeoGebra. Teachers wanted to know how to delete the trace or why it couldn’t be saved with the rest of the construction.
- *Other features*: 11 helper reports pertained to other GeoGebra features. Participants required the helpers’ support when having to undo mistakes. Corrections can easily be done using the *Undo* button, but many teachers seemed to forget this. Others wanted to get additional information about the *Construction protocol* and *Navigation bar* of GeoGebra. Finally, participants had difficulties inserting or editing text in the drawing pad. Dynamic text especially seemed to be challenging since several participants forgot to create the objects first before referring to them in the dynamic text.

Algebraic input and commands: In this category the helper feedback concerning algebraic input and use of pre-defined commands in GeoGebra is summarized.

- *Input syntax*: In 28 reports helpers stated that participants had difficulties when trying to enter algebraic expressions into the input field. Most issues dealt with mixing up upper and lower case letters and using variables within the expressions that didn’t exist as objects yet. Additionally, forgetting to use a space or asterisk when multiplying two variables was a very common mistake. Another issue the helpers needed to address was that participants didn’t read the error message that appeared when they made a mistake. Therefore, they never knew what the problem was and how to solve it, but relied on the helpers to find out what was wrong.

Learning to read the error messages and trying to figure out the mistake on their own, are probably the most important things to teach the participants in this case. Additionally, the differences between upper and lower case letters in GeoGebra need to be addressed more clearly (e.g. points are labelled with an upper case letter, while the variable x within a function or equation always needs to be lower case). In order to help users with correcting mistakes related to

algebraic input, GeoGebra could provide more specific error messages that point out the actual mistake more clearly. Additionally, a selection of possible syntax corrections could be offered allowing users to choose the one they intended to enter in the first place.

- *Use of commands*: 9 reports pertained to the use of commands in GeoGebra, whereby most of the issues dealt with mixing up the name of the newly created object and the names of parent objects to which the command was applied. For example, in the expression $V = \text{Vertex}[p]$, V is the name of the vertex while p refers to an already existing parabola.

Thus, the syntax of using commands needs to be explained in more detail. Moreover, the importance of creating parent objects before using their names within commands as well as the difference to naming a newly created object need to be discussed and tried out with the participants.

Chapter 8

Implementation of Research Outcomes

8.1 Summary of Research Outcomes

After analyzing the data collected during the four introductory workshops for GeoGebra, the auxiliary research questions specified in section 4.4.1 are answered and a summary of potentially relevant results for the development of more successful GeoGebra instructional materials is given. Finally, the key research question is answered in this section.

8.1.1 Answers to Auxiliary Research Questions

Introductory Workshops

1. *Are design, content, and difficulty level of the introductory workshops appropriate for secondary school teachers?*

Considering the rather low difficulty ratings for workshop activities between $\bar{x} = 1.02$ for the subjectively easiest task (workshop I, activity 1) and $\bar{x} = 2.05$ for the most challenging task (workshop II, activity 4) on a scale from 0 ('very easy') to 5 ('very difficult'), the difficulty level of the introductory workshops seemed to be appropriate for secondary school mathematics teachers. The majority of participants enjoyed the workshops in general and liked the variety of contents covered. The content ranged from geometric constructions, to experimenting with linear equations, and to working with functions in GeoGebra. Although the home exercises were a little more challenging for some of the teachers, in general the participants didn't seem to be overwhelmed and managed to get along quite well with GeoGebra when using it at home. Furthermore, the combination of different teaching methods and hands-on activities seemed to appeal to the participants and provided the necessary diversion to keep the participants focused and motivated (see sections 5.2 and 5.3).

GeoGebra in General

2. *How do teachers experience the introduction to GeoGebra and what kind of feedback do they give concerning its usability?*

Overall, the participants seemed to enjoy the introduction to GeoGebra and gave mostly positive feedback about the software itself. They characterized GeoGebra as user friendly, easy and intuitive to use, and potentially helpful for teaching mathematics in secondary schools. They especially liked the versatility of the software, as well as the potential to create appealing activities and instructional materials for students (see section 5.2.2).

Subjective Difficulty Rating of GeoGebra Tools

3. *Do users tend to subjectively rate GeoGebra's dynamic geometry tools to be of different difficulty levels when they are introduced in a workshop?*

Although the average difficulty ratings of dynamic geometry tools on the day of their introduction didn't exceed 1.8 for any of the tools, which is within the lower half of the provided scale from 0 ('very easy') to 5 ('very difficult'), differences related to the complexity and use of certain tools could be found. Nevertheless, reusing the tools on another workshop day, as well as additional practice and increasing familiarity with GeoGebra, caused the difficulty ratings to decrease. Participants placed all tools within the group of 'easy to use' tools when they were rated for a second time regardless of their initial subjective difficulty ratings (see section 6.1).

Impact of Workshop Activities on Tools Ratings

4. *Do activities used to introduce dynamic geometry tools have impact on their subjective difficulty ratings?*

Concerning the impact of workshop activities on the subjective rating of introduced GeoGebra tools, the evaluation results showed a strong correlation between the ratings and activities, indicating that there actually is a connection between them. Although the direction of this influence can't be determined for sure, the participant's open feedback as well as helpers' observations during the workshops indicate, that complex activities have negative influence on the subjective difficulty ratings of introduced tools. Although this needs to be examined more closer, a reduction of the content's difficulty level for activities used to introduce new tools (see section 6.3.3) is suggested.

Complexity Classification of Dynamic Geometry Tools

5(a). *Is it possible to classify GeoGebra's dynamic geometry tools under groups of common characteristics that determine their general difficulty levels?*

As shown in section 6.3.1, the analysis of introduced GeoGebra tools revealed common characteristics which facilitated definition of five complexity criteria for dynamic geometry tools. After examining the number and type of required existing objects, the number and order of actions necessary, as well as required keyboard input, all GeoGebra tools could

be classified and assigned to three different complexity level groups. This categorization now provides a basis for the development of more successful introductory materials. By taking into account the complexity of each tool, appropriate methods of introduction can be selected. This result could potentially make it easier for novices to learn how to use a new tool and could help to prevent unnecessary impediments for participants.

5(b). *Can the same classification criteria that determine the difficulty level of GeoGebra tools also be applied to the construction tools of other dynamic geometry software packages?*

The complexity criteria established in order to classify GeoGebra's dynamic geometry tools also seem to be effective for the tools of Cabri II Plus and Geometer's Sketchpad (see section 6.5). Although the basic use and selection of provided tools differ from GeoGebra, the tools of these two programs were successfully classified thereby allowing a determination of their general difficulty levels. By detecting 'difficult to use' tools which potentially could cause additional problems for novices, the introduction process for Cabri and Sketchpad could also be modified correspondingly in order to facilitate the first contact of teachers with dynamic geometry software in general.

GeoGebra Features and Algebraic Input

6. *Do GeoGebra's features, algebraic input, or commands cause additional difficulties for the introduction of GeoGebra?*

Although no differences between the subjective difficulty level of pure geometry tasks and activities involving algebraic input could be found for workshop activities, participants tended to spend more time on home exercises that required algebraic input or the use of commands. Additionally, participants reported difficulties with the syntax of algebraic input, as well as the use of commands whose use was subjectively rated more difficult than the use of dynamic geometry tools (see section 7.1).

Concerning GeoGebra's features, participants regarded features with more options (e.g. point capturing, types of labels) to be more challenging than those that simply can be turned on and off. Although the way of accessing the features in general didn't cause additional problems, the right-click necessary to open the *Context menu* confused some teachers, and therefore caused difficulties mainly among the MS Windows users (see section 7.2). Thorough explanations and practice time need to be given in order to facilitate the introduction of features, algebraic input, and commands, and to prevent the foregoing common problems.

Impact of External Variables

7. *Do external variables such as math content knowledge, computer literacy, or the use of a touchpad, influence the subjective difficulty rating of GeoGebra, its tools, or its features?*

As shown in section 7.3, external variables such as gender, age, teaching experience, mathematics content knowledge, subjectively rated computer skills, or operating system

seem not to have any influence on the difficulty ratings of workshop activities, GeoGebra tools, algebraic input, features, or commands. Only the use of a touchpad instead of a computer mouse proved tricky and caused additional difficulties which influenced the subjective difficulty ratings of the participants. Since this can be easily prevented by providing computer mice for the participants, GeoGebra's user friendly design and ease of use seem to make it an appropriate tool for all evaluated groups of workshop participants. This is especially important considering its international user community.

Frequent Problems During Introductory Workshops

8. *Which difficulties, problems, and questions occur most often during an introductory workshop?*

By analyzing the helper reports (see section 7.4), the difficulties, problems, and questions of participants could be categorized. Most assistance was required in the context of the construction process of geometric figures, general concepts related with dynamic mathematics software, the general use of dynamic geometry tools, as well as proper syntax for algebraic input and the use of commands. Since most of the issues were related to topics covered by the presenter earlier, detailed handouts for all workshop sections, as well as a collection of tips and tricks for the basic use of GeoGebra could potentially help to prevent most of them.

8.1.2 Results Relevant for Designing Introductory Materials

This section emphasizes those results of the GeoGebra introductory workshop evaluations that are potentially most relevant for the development and design of more successful introductory materials. These materials could include, by way of example, a workshop guide for presenters who want to introduce GeoGebra to novices.

- Workshops should possess a **flexible design** that allows the presenter to respond to the participants' needs and interests by adapting the content covered. Special mathematical topics of different difficulty levels could be offered in addition to the basic GeoGebra introductory workshop allowing extension of the workshop length and concentration on certain topics.
- Since the **technical environment** and **computer skills** of participants are different for each introductory workshop, GeoGebra should be installed in the beginning of the workshop in order to familiarize participants with this process. If an Internet connection is accessible, GeoGebra's WebStart version should be used since it doesn't require special user permissions from a computer and can easily be used in a computer lab as well. In this case, also, all necessary files (e.g. picture files) can be downloaded from the web page and saved on the desktop in order to ensure easy access during the workshop activities. If no Internet connection is accessible, participants could either be asked prior to the workshop to install the software on their notebook computers by

following step-by-step instructions, or GeoGebra could be installed after distributing the corresponding files via USB drives or CDs.

In order to prevent other common computer issues, participants should create a folder on their desktop which can be used to save all created GeoGebra files so as to facilitate file location if they are reused later on. Additionally, the process of saving a file should be clarified and step-by-step instructions should be provided on a handout.

- The presenter should encourage participants to use a **computer mouse** with their notebook computer in order to operate GeoGebra. Therefore, a set of computer mice should be brought along by the presenter which can be borrowed by participants. Additionally, the importance of using a mouse should be explained together with its advantages. Teachers should know that touchpad issues could potentially cause unnecessary difficulties for their students. Furthermore, the differences between a left and right click need to be discussed thoroughly in order to prevent frustration when trying to open the *Context menu* of GeoGebra.
- The amount of **mathematical content covered** in the workshop shouldn't overwhelm the participants. Therefore, small portions of information should be given, followed by some practice time allowing participants to process the new knowledge and for practicing their new skills. During these practice blocks, a pool of activities could be offered containing tasks of different difficulty levels ranging from basic applications to challenge tasks. Thus, participants could select activities of their interest that meet their individual skills and needs. If desired, participants could also work in pairs, giving them the opportunity to discuss experiences and problems with a colleague as well as to deepen their knowledge about the use of GeoGebra.
- Although the content of the introductory workshops seemed to be appropriate, **mathematical concepts** and **construction processes** need to be clarified prior to the corresponding activity in order to prevent additional difficulties when using GeoGebra for the first time. Properties of dynamic figures should be discussed with the participants in combination with teaching them how to select appropriate tools for every construction step. Additionally, the construction protocol could be used to explain why the order of construction steps could potentially be important, as well as to teach the concepts of 'parent' objects and dependencies between objects.
- In order to **introduce new dynamic geometry tools**, less complex activities should be used. The number of tools introduced in each activity needs to be kept rather low and the participants should be allowed to try new tools prior to working on the activity. No tool should be introduced along the way since this might cause additional difficulties. Tools from the 'General tools' toolbox also need to be explained and tried in order to familiarize participants with their use. Activities that combine algebraic input and the introduction of new tools need to be treated with special care since they are more complex and tend to cause additional troubles. In order to

emphasize common characteristics as well as differences between similar tools (e.g. *Perpendicular line* and *Parallel line*), such pairs of tools should be introduced during the same activity with their relation explained prior to actually using them within the construction.

- New dynamic geometry tools always should be introduced according to their **complexity group classification**. ‘Easy to use’ tools can be explored and tried by the participants in order to foster their independent use of GeoGebra at home. The use of tools assigned to the ‘middle’ group could be demonstrated by the presenter and their characteristics explained (e.g. order of clicks, types of objects required) while participants are encouraged to simultaneously work along. Finally, ‘difficult to use’ tools should be thoroughly discussed prior to actually using them in order to prepare participants for different actions required for successful application of the tools. The presenter needs to pay special attention to required keyboard input allowing time for practicing those tools afterwards.
- Participants need to learn how to **properly activate a tool** whose icon is already shown in the toolbar, as well as how to **select appropriate tools** for every construction step. Additionally, **reading the toolbar help** in order to find out how the selected tool can be operated needs to be emphasized throughout the workshop in order to prepare participants for using GeoGebra independently. The connection between tools of the same toolbox should be explained so as to facilitate finding the desired tool within the toolbar.
- **Hands on activities** and **discussions** are important parts of an introductory workshop and need to be emphasized. Additionally, a variety of best **practice examples** should be presented in order to familiarize the teachers with ways of using GeoGebra with their students thereby helping them to transfer their new skills to their classrooms. GeoGebra’s potential for facilitating mathematics teaching, as well as for helping students to discover and better understand mathematical concepts should be discussed with the teachers. Furthermore, benefits of creating one’s own instructional materials should be discussed in order to help them to become less dependent upon the textbooks, foster their own creativity, as well as adapt their teaching methods to their students’ individual needs.
- Although a slow pace is potentially difficult to implement, presenters should never rush. They should take time to **thoroughly explain** and give detailed instructions. Additionally, each action presented should also be tried by the participants. During an introductory workshop, **different presentation methods** should be implemented: apart from allowing participants to work along with the presenter, constructions could also be presented first and then redone by participants. Participants should be encouraged to take notes during the presentation which potentially would help them to repeat the activity and prevent difficulties and problems. In order to

foster the teachers' independent use of GeoGebra, they should also have opportunities to figure out some tasks on their own or with a colleague, and they should be encouraged to present their solutions afterwards.

- Since in a typical workshop, the presenters might be on their own or supported by one assistant, certain **directives** should be discussed with the participating teachers. If problems occur, a colleague should be asked for help before addressing the assistant or presenter. If a question of general interest is posed, it should be discussed with all participants in order to prevent the assistant or presenter from having to give the same answer multiple times. Furthermore, the presenter should include discussion times and pause frequently. This will allow for questions to be posed and potential difficulties to be solved.
- **Proper documentation** should be provided for introductory workshops, including...
 - detailed handouts
 - construction protocols for geometric constructions
 - a summary of all GeoGebra tools, their assignment to certain toolboxes, as well as explanations on how to use them
 - a description of menu items and corresponding features (including explanations for features with multiple options)
 - detailed information about the syntax of algebraic input and commands including an overview of the most common mistakes and error messages
 - instructions on how to resize images prior to inserting them into GeoGebra
 - detailed instructions on how to work with MS Word in combination with GeoGebra (e.g. inserting and resizing images, creating a table to align contents, inserting formulas), in order to create notes, worksheets, tests or quizzes

Furthermore, all workshop materials should be provided online as well so as to allow participants to download and review them at home.

- Special attention and explanations should be given concerning the following **GeoGebra features**.
 - How to select an already existing object in order to use it for the creation of new objects. An object highlights when the pointer is moved close to it and the shape of the pointer changes, indicating that a click would select the object.
 - How to select objects, which lie on top of each other and therefore can't be selected directly. Instead, a dialog window appears listing all objects in this position, allowing user selection of one of them.

- How to select several objects at the same time: (i) using the selection rectangle, (ii) holding the *Ctrl*-key depressed while selecting the objects, or (iii) holding the *Shift*-key depressed while selecting the first and last desired object from a list (e.g. algebra window, *Properties dialog*).
- How to undo mistakes in a step-by-step fashion by using the *Undo* button, as well as different ways of deleting objects: (i) tool *Show / Hide object*, (ii) right-clicking an object and selecting ‘Delete’ from the *Context menu*, or (iii) selecting an object and pressing the *Delete*-key.
- How to prevent the creation of unwanted objects when trying to move an already existing object with the mouse. The importance of ‘back to *Move* mode’ needs to be mentioned frequently throughout the workshop in order to avoid frustration among the participants.
- How to distinguish and create free and dependant objects in GeoGebra: e.g. position in the algebra window, different color). Characteristics of dependant objects need to be discussed and assigning objects to the group of auxiliary objects needs to be practiced.
- How to avoid the most common mistakes involving the syntax for algebraic input as well as the use of commands in GeoGebra: e.g. issues with upper and lower case letters, objects need to be defined prior to using their names as variables within algebraic expressions and commands, naming an object versus using its name within a command’s brackets.

Additionally, the most common error messages caused by such mistakes should be listed on a handout in order to help teachers to understand why they appear and how to fix the mistake.

- How to deal with pop-up windows that appear. Since many novices tend to close such a window without reading its content, this issue needs to be addressed while encouraging teachers to read these messages which potentially might help them find a solution for their problem.
- How to distinguish between the *Properties dialog* for usual objects and for the drawing pad, as well as how to access the *Properties dialog* over the menu bar.
- How to deal with the *Trace on* feature and its characteristics: e.g. the trace doesn’t appear in the algebra window, the trace of an object can’t be saved, the trace is deleted whenever the graphics are refreshed.
- How to prevent participants from mixing up the *Rename* and *Redefine* features by introducing the former using the ‘fast-renaming’ option at the beginning. Later on, when feature *Redefine* is introduced too, the differences between those two features need to be thoroughly explained.
- How to insert images into GeoGebra’s graphics window using appropriate image files provided on the webpage or off-line by the presenter. Ways of resizing

images prior to inserting them into GeoGebra need to be discussed, as well as issues with finding images on the Internet need to be clarified.

- How to increase the font size within the GeoGebra window for presentation purposes.

8.1.3 Answer to the Key Research Question

Is it possible to identify common impediments that occur during the introduction process of dynamic mathematics software as well as to detect those especially challenging tools and features of the software GeoGebra in order to (a) provide a basis for the implementation of more effective ways of introducing dynamic mathematics software to secondary school mathematics teachers and (b) to design corresponding instructional materials for technology professional development?

During the evaluation of a series of GeoGebra introductory workshops a variety of difficulties and frequent impediments that arose during the introduction process of dynamic mathematics software were documented. By analyzing the data collected as well as the feedback given by participants and workshop helpers these impediments could be organized and summarized with regard to the auxiliary research questions posed in section 4.4.1. Based on the subjective difficulty ratings and a thorough analysis of the functionality of GeoGebra tools, complexity criteria for dynamic geometry tools could be established allowing determination of the general difficulty level of such tools while identifying those which are especially challenging for novices. Additionally, the data collected proved useful for assessing difficulties related to the use and functionality of certain GeoGebra features.

In order to prevent the most common difficulties in future technology workshops with GeoGebra, possible reasons for their occurrence as well as suggestions on how to avoid them during introductory workshops are given in this thesis. By transforming them into guidelines for presenters and implementing these guidelines in future GeoGebra workshops as well as incorporating them into the design of accompanying instructional materials, the software could potentially be introduced more effectively to mathematics teachers during future technology professional development events.

8.2 Design of New GeoGebra Introductory Workshops

Based on the outcomes of the study described in this thesis which were summarized in section 8.1.2, documentation for a new and potentially more successful GeoGebra introductory workshop is currently being designed. Its main goals are the prevention of common difficulties and impediments that occur during the introduction of new software to teachers and to facilitate their first contact with GeoGebra in order to prepare teachers for its effective integration into their teaching practices.

8.2.1 GeoGebra Introductory Workshop Guide

The new *GeoGebra Introductory Workshop Guide* is a detailed document for presenters who are hosting a GeoGebra introductory workshop for secondary school teachers. On the one hand, this workshop guide is intended to set a certain quality standard for professional development with GeoGebra. Thereby, findings of the GeoGebra workshop evaluation as well as research outcomes from other sources are implemented in order to make the first contact with the software as easy as possible for teachers. On the other hand, this document covers all basic features of GeoGebra and contains best practice examples as well as a selection of practice activities in order to ensure that beginners get a sufficient overview of the potential and application possibilities of the software.

Since the GeoGebra Introductory Workshop Guide will be revised according to a formative evaluation process and feedback of the presenters and participants, it is not included in the appendix of this thesis. Instead the document will be available online¹.

Structure and Content

The *GeoGebra Introductory Workshop Guide* is designed to provide content for a full-day GeoGebra workshop of about 6 hours length. The document consists of eight topic blocks covering the basic use and main features of GeoGebra, and four practice blocks that allow participants to select activities from a provided pool in order to practice new skills by means of individually meaningful tasks. Each of these twelve blocks is designed to take about 30 minutes. However, the time frame can be flexibly adapted to the content knowledge, computer skills, and individual needs of the participating mathematics teachers.

Topic 1. What is GeoGebra: Participants are provided with background information about GeoGebra and learn how to install the software on their computers. Installation options are discussed (installer versus WebStart version) and all files necessary for the workshop are transferred to each computer (e.g. construction files, picture files). Additionally, participants have the opportunity to draw figures with GeoGebra and to try out a selection of geometry tools on their own. Finally, teachers learn how to save GeoGebra files on their computers.

Topic 2. Basic Geometric Constructions: Participants learn about the differences between drawings and constructions in a dynamic geometry environment and experience the potential of the drag-test. A selection of geometric construction tools is introduced by means of guided constructions. Additionally, participants learn how to use GeoGebra's *Properties dialog* in order to change properties of objects and enhance their constructions.

Practice Block I: Participants can practice their new skills by means of a pool of specially designed activities from which they can pick tasks that meet their individual

¹GeoGebra Introductory Workshop Guide: www.geogebra.org/en/wiki/index.php/Workshop_materials

interest and desired technical and mathematical difficulty level. Detailed instructions, additional challenge problems, and solutions are provided for each activity. While basic tasks only require the use of already introduced tools, advanced tasks might involve exploring the use of new tools and they are usually mathematically and technically more demanding. In this practice block participants have the opportunity to explore geometric constructions and practice applying different dynamic geometry tools in GeoGebra.

Topic 3. Inserting Pictures and Text: Participants learn how to insert pictures into GeoGebra and use them either as background images or as active part of a construction. Thereby, necessary image files are provided by the presenter in order to prevent additional problems for the participants which could arise from looking for suitable pictures on their computers or the Internet. Additionally, teachers learn how to insert static and dynamic text into the graphics window of GeoGebra and are introduced to certain transformation tools.

Topic 4. Basic Algebraic Input and Commands: Participants are introduced to the algebra window and input field of GeoGebra and learn how to enter algebraic input and commands in order to create mathematical objects by using the keyboard instead of the mouse. The syntax needed in order to properly communicate with GeoGebra as well as most common error messages connected to keyboard input are discussed. Furthermore, participants learn about the bidirectional connection between algebraic and graphical representations of the same mathematical objects and about ways of influencing both display formats.

Practice Block II: Participants can practice their newly developed skills by means of specially designed activities. Again, all activities are provided with detailed documentation as well as additional challenge tasks and differ in terms of difficulty level of content and GeoGebra use. In this practice block special focus is given to inserting images and text into GeoGebra constructions, as well as using the keyboard for algebraic input and commands in order to create dynamic figures.

Topic 5. Creating Static Instructional Materials: Participants learn how to export dynamic figures as static pictures which can be inserted into other documents (e.g. word processing documents) in order to create printable instructional materials or presentations. Additionally, teachers learn how to enter different types of functions in GeoGebra and how to integrate them into engaging exploration activities for students.

Practice Block III: Participants can practice creating static instructional materials by means of several different activities of varying difficulty level and mathematical content. They are encouraged to create printable mathematical games for their students and practice their skills in the context of using word processing software in combination with pictures exported from GeoGebra.

Topic 6. Conditional Visibility of Objects: Participants learn how to use checkboxes in order to show and hide single or groups of objects of a GeoGebra construction. They are introduced to the ‘Advanced’ tab of the *Properties dialog* which allows users to determine conditions for the visibility of an object. Additionally, information about the meaning, usage, and potential of Boolean variables and ‘if’-statements in GeoGebra is presented.

Topic 7. Creating Dynamic Worksheets: Participants learn how to export their constructions as dynamic worksheets in order to be able to create web-based interactive activities for their students. Quality issues of dynamic worksheets are discussed, and teachers learn how to design them in a reasonable way that fosters students’ learning of mathematics. Additionally, tips and tricks for dealing with different types of files, as well as options to provide dynamic worksheets to students and other teachers either online or on local storage devices are discussed.

Practice Block IV: Participants have the opportunity to practice the creation of dynamic worksheets by means of specially designed tasks. Again, they can pick from a selection of different mathematical topics and difficulty levels. In this practice block, skills from all previously presented theory blocks can be practiced and enhanced.

Topic 8. GeoGebraWiki and User Forum: Participants get information about the GeoGebraWiki, which is a pool of free instructional materials, as well as about the GeoGebra User Forum, which provides online support for GeoGebra users. Collaboration opportunities as well as potential benefits of an international user community are discussed and information about additional support possibilities beyond the workshop itself is provided.

Differences to Prior GeoGebra Workshops

Compared to the GeoGebra workshops that were evaluated in the context of this thesis (see chapter 4), the new *Introductory Workshop Guide* for GeoGebra features some important improvements and modifications concerning structure and flexibility, as well as mathematical and technical content of the workshops.

Practice opportunities: The newly designed GeoGebra introductory workshop contains special practice blocks allowing participants to spend time practicing new skills and processing all the technical and mathematical information given in the previous topic blocks. Participants can pick from a provided pool of prepared and well-documented activities according to their computer skills, individual interests, and mathematical content knowledge. Many activities involve prepared dynamic worksheets and GeoGebra files allowing teachers to explore GeoGebra’s potential for students’ learning and understanding of mathematics and providing best practice examples for a successful integration of GeoGebra into teaching practices.

Documentation and Handouts: In the newly designed GeoGebra introductory workshop, detailed handouts for participants are provided which are based on the *GeoGebra Introductory Workshop Guide* for presenters. They contain detailed step-by-step instructions for all activities of topic and practice blocks. Although these documents are intended to accompany a GeoGebra introductory workshop and to support subsequent independent use of the software at home, they contain a sufficient amount of additional information and instructions to be also used for a self-dependent exploration of GeoGebra by teachers.

Transfer: By increasing the amount and variety of best practice examples within the topic and practice blocks of the new GeoGebra introductory workshop, participants can explore a wider range of application possibilities for the software. Teachers can experience how their students could explore mathematical concepts by means of ready-to-use instructional materials such as dynamic worksheets or worksheets on paper. By letting teachers experience the potential benefits for students and providing best practice examples, ready-to-use materials, as well as ideas about how GeoGebra could effectively be integrated into a ‘traditional’ mathematics classroom, the transfer and integration process of the software into everyday teaching could be facilitated and encouraged.

Relevance: The flexible structure of the new GeoGebra introductory workshop, the variety of content covered, as well as the integration of best practice examples and ready-to-use instructional materials could potentially increase the relevance of the workshop content for individual teachers. By meeting teachers’ actual needs and providing them with interesting and engaging materials that are relevant for their individual teaching situation and environment, the integration of this new software into classrooms could be supported and teachers’ efficient use of GeoGebra could potentially be fostered.

Flexibility of structure: Due to the flexible structure and design of the workshop as a series of content topic and practice blocks, the various needs of GeoGebra’s international user community can be served more easily. The new workshop materials (e.g. GeoGebra Introductory Workshop Guide for presenters, handouts for participants) can be translated to other languages and their content can be adapted to the local needs of mathematics teachers in other countries by adapting or replacing certain activities with more suitable ones that meet the objectives of local mathematics curricula.

Information for presenters: In order to enhance the GeoGebra introductory workshop over time, even if it is given by different presenters, detailed information about potential difficulties and problems that occur frequently during introductory workshops is provided to the presenters. In this way, participants can benefit from the experiences obtained during previous workshops which could potentially improve their first experiences with the software by preventing them from making the most common

beginner-mistakes and by fostering a more successful and satisfying first contact with GeoGebra.

8.2.2 Additional Workshop Documentation for Participants

In order to support workshop participants during and after a GeoGebra introductory workshop, they will be provided with additional documentation in form of handouts that can be used to review content covered during a workshop or to look up solutions for occurring difficulties and problems. Main objective for the development of these materials is to foster an independent use of the software and to facilitate its introduction and further use for mathematics teachers, as well as its successful integration into a teaching and learning environment. All materials described below will be available online².

General Workshop Document

During a GeoGebra introductory workshop, participants will be provided with a special version of the *GeoGebra Introductory Workshop Guide*. Although it won't contain the additional information meant for the presenter (e.g. time frames, comments), it will give a complete overview about the contents of the workshop and will contain detailed instructions for all workshop activities.

Additionally, this document will provide tips and tricks related to the basic use of GeoGebra, and it will address the most common difficulties and problems that arise during the introduction of the software. Although this document is intended to provide additional support during or after an introductory workshop, it will also be appropriate to learn how to use GeoGebra on one's own without having additional instruction.

Dynamic Geometry Tools Overview

This handout will give an overview of all available dynamic geometry tools of GeoGebra and explain how each tool can be applied. Information about the corresponding difficulty level of each tool, which is based on the complexity criteria for dynamic geometry tools (see section 6.3.2), will be given and special requirements of the tools will be listed (e.g. required existing objects, relevant order of clicks, required input). Additionally, the tools' organization in toolboxes as well as proper ways of activating tools will be explained and tips and tricks concerning the use of dynamic geometry tools will be provided.

Algebraic Input and Commands Overview

This handout will give an overview of the use of algebraic input and commands in GeoGebra. Information about the required syntax will be given and most common error messages concerning algebraic input and commands will be explained. This document is intended to facilitate the use of keyboard input for teachers who are not used to dealing with computer

²Additional workshop documentation: www.geogebra.org/en/wiki/index.php/Workshop_materials

algebra systems and to help teachers to overcome initial difficulties they might experience when dealing with algebraic input and commands.

GeoGebra's Menu Items and Features Overview

This handout will explain the menu items available in GeoGebra and give information about different settings of the software. The document is intended to give an overview of available setting options that potentially could make it easier or more comfortable for teachers to use GeoGebra with their students. Additionally, special features like the *Properties dialog* or *Context menu* will be described and tips and tricks for the handling of GeoGebra will be given in order to prevent the most common difficulties related to GeoGebra features.

Creating Static Instructional Materials

Since most of the issues that arise during the creation process of static instructional materials with a text processing software in combination with GeoGebra are not directly related to the dynamic mathematics software, they can't sufficiently be discussed and explained during a usual introductory workshop. This handout will contain additional information about the creation of static instructional materials with special focus on how to operate text processing software like MS Word. Formatting issues will be discussed and the insertion of pictures will be explained in order to support teachers who are not sufficiently familiar with using MS Word and to make it easier for them to create their own instructional materials.

8.2.3 Design Guidelines for Dynamic Worksheets

Although today's Internet already provides a large number of freely available interactive instructional materials for mathematics teaching and learning (see section 3.3.3), their actual creation used to be reserved for people with high computer skills or even programming experience. Since GeoGebra provides an easy way of creating dynamic worksheets, many teachers will soon be able to create their own interactive learning environments that exactly meet their students mathematical content knowledge, skills, and needs.

The GeoGebraWiki³ is a pool of free dynamic worksheets which were created by teachers from all around the world who were willing to share their materials with the GeoGebra user community. However, the quality of these dynamic worksheets varies not only in terms of mathematical content and implementation of technical details, but also concerning usability, interactivity, and overall design. Since the number of available instructional materials on this platform is increasing steadily it becomes more and more difficult for teachers to pick from the variety of prepared ready-to-use materials those that meet their students' needs and foster effective learning.

Research indicates that the design of e-learning materials potentially can influence the learning progress of students [Clark and Mayer, 2003] in both negative as well as positive

³GeoGebraWiki: www.geogebra.org/en/wiki

ways. Since most of the materials on the GeoGebraWiki are subject to the *Creative Commons License*⁴ teachers are allowed to use the original materials with their students as well as to modify the dynamic worksheets under certain conditions in order to meet their students individual needs [Hohenwarter and Preiner, 2007a, pp. 8]. Knowledge about the implementation of e-learning principles and their importance for effective student learning could possibly help teachers when creating their own or modifying existing dynamic worksheets in order to improve their usability and reduce the cognitive load for students who will use these materials.

Common Design Issues and Useful e-Learning Principles

In order to set a certain quality standard for dynamic worksheets and make it easier for teachers who want to create their own or modify existing interactive learning environments that were created with GeoGebra, corresponding instructional materials will be developed and provided online as well as during future GeoGebra workshops. These materials are intended to support teachers with the task of creating instructional materials that foster active and student-centered learning experiences, mathematical explorations, and discovery learning in order to improve students' understanding of mathematical concepts. They will provide information about cognitive load theory [Sweller, 1988], e-learning principles [Clark and Mayer, 2003], and their implementation in the design of interactive learning environments like dynamic worksheets [Hohenwarter and Preiner, 2008].

By formatively evaluating dynamic worksheets created by mathematics teachers who participated in the Math and Science Partnership project at Florida Atlantic University (see section 4.1.1) in Fall 2006 and throughout 2007, frequently arising difficulties and impediments concerning the design and quality of these interactive materials were identified and summarized. The most common problems were not related to technological difficulties but pertained to design issues like the layout, conception of the dynamic figure, as well as phrasing of explanations and tasks for students [Hohenwarter and Preiner, 2008, p. 3].

Additionally, some of the e-learning principles stated by Clark and Mayer [Clark and Mayer, 2003] promised to be useful for the design of dynamic worksheets and support the experiences gathered with teachers.

Multimedia Principle: "Use words and graphics rather than words alone" [Clark and Mayer, 2003, p. 51]. Providing the graphical representation in addition to textual description or algebraic representation of mathematical objects potentially can foster students' learning. In GeoGebra this e-learning principle is directly implemented by offering different representations of the same mathematical objects and connecting them dynamically. This feature can easily be integrated into the design of dynamic worksheets in order to foster more effective student learning experiences [Hohenwarter and Preiner, 2008, p. 4].

Contiguity Principle: "Place corresponding words and graphics near each other" [Clark and Mayer, 2003, p. 67]. By directly inserting dynamic text into the drawing

⁴Creative Commons License: www.geogebra.org/en/cc.license/cc_license.htm

pad and by displaying labels of objects that include their names as well as values, this e-learning principle can be implemented rather easily in GeoGebra constructions and dynamic worksheets. Since labels are by default and dynamic texts can manually be attached to their corresponding objects, they automatically adapt to movements and stay close to them [Hohenwarter and Preiner, 2008, p. 5].

Coherence Principle: “Adding interesting material can hurt learning” [Clark and Mayer, 2003, p. 111]. Although GeoGebra allows users to insert pictures into the graphics window and to use them either in the background or as active part of the construction, teachers need to be careful when using this feature within dynamic worksheets. Decorating pictures can enhance the layout of instructional materials but they are often distracting and make it more difficult for students to focus on the mathematical task [Hohenwarter and Preiner, 2008, p. 6].

Personalization Principle: “Use conversational style . . .” [Clark and Mayer, 2003, p. 131]. Phrasing explanations and tasks in personal style potentially makes them easier to understand and allows teachers to directly address students. This can increase their motivation to put more effort into solving given tasks and reaching certain objectives [Hohenwarter and Preiner, 2008, p. 6].

Design Guidelines for Dynamic Worksheets

The following design guidelines for dynamic worksheets are based on common design mistakes and the e-learning principles mentioned above. Being implemented, these guidelines could possibly foster more effective student learning by reducing the cognitive load in the design of instructional materials. Nevertheless, further research needs to be conducted in order to assess if the design of dynamic worksheets really affects the learning progress of students and whether or not interactive instructional materials that are designed according to these guidelines are more successful than other technology-based learning environments.

The following four design guidelines are related to the *general layout* of dynamic worksheets [Hohenwarter and Preiner, 2008, p. 8]. They are intended to support a user-friendly layout that fosters successful mathematical learning.

Avoid Scrolling: The entire dynamic worksheet should fit on one screen in order to prevent students from having to scroll between the tasks and the dynamic figure (see *Contiguity Principle*). Today’s usual screen size constrains the size of the dynamic worksheet, which can be adjusted using an HTML editor (e.g. NVU⁵). If this is not possible, teachers should consider to break the dynamic worksheet into several pages and to connect them using hyperlinks.

Short explanation: In order to introduce students to a new dynamic worksheet and give them an overview of the mathematical concept involved, a short explanation should

⁵NVU: <http://www.nvu.com>

be included at the beginning of the dynamic worksheet. The text should be short and phrased in personal style (see *Personalization Principle*).

Few tasks: Tasks and questions will foster students' active use of a dynamic worksheet. They should be placed close to the dynamic figure and their number should be limited to three or four in order to avoid scrolling (see *Contiguity Principle*). If teachers want to add more tasks to the same dynamic figure, they should consider breaking the dynamic worksheet into several pages and connect them with hyperlinks in order to increase their usability.

Avoid distractions: A dynamic worksheet should exclusively contain objects that are relevant for the mathematical content and the objectives students are supposed to reach. Unnecessary background images, purely decorative images, as well as other potential distractions should be omitted when designing dynamic worksheets (see *Coherence Principle*).

The next set of design guidelines is related to the *dynamic figure* which is a vital part of a dynamic worksheet [Hohenwarter and Preiner, 2008, pp. 9].

Interactivity: A dynamic worksheet should allow plenty of freedom to explore relations between mathematical objects and to discover mathematical concepts actively. Teachers should allow as much interactivity in the dynamic figure as possible by including moveable objects and possibilities for modifications that are relevant for reaching the objective of the dynamic worksheet (see *Coherence Principle*).

Ease of use: Dynamic worksheets should be easy to use and should not require many instructions and explanations. By changing the appearance of moveable objects (e.g. different color or size) and fixing objects that are not supposed to be changed, students will get less distracted and can spend more time exploring the mathematical concepts intended by the teacher (see *Coherence Principle*).

Size matters: Although a dynamic figure should be big enough to allow for manipulations and mathematical experiments it still needs to fit on one screen and leave sufficient space for explanations and tasks on the dynamic worksheet (see *Contiguity Principle*).

Dynamic text: Dynamic text could be included into the graphics window in order to prevent students from being distracted by the algebra window which could be hidden (see *Coherence Principle*). Additionally, labels and dynamic text should be placed close to their corresponding mathematical objects (see *Contiguity Principle*).

Avoid static text: Static text should not be placed within the dynamic figure in order to avoid cluttering the interactive applet (see *Multimedia Principle* and *Coherence Principle*). Instead, explanations and tasks can be placed next to the dynamic figure on the surrounding web page.

First appearance: When opening a dynamic worksheets, all labels and dynamic text should be readable, and all relevant objects should be clearly visible. GeoGebra allows users to rearrange labels and text as well as to change properties of objects in order to enhance the layout of a dynamic figure. This makes it easier for teachers to create materials with an appealing first appearance.

Finally, there are some design guidelines that refer to the *explanations and tasks* of a dynamic worksheet [Hohenwarter and Preiner, 2008, pp. 10]. They could potentially make it easier for students to understand instructions on the dynamic worksheets and help them to reach the mathematical objectives intended by the teacher.

Short, clear, and personal style: Explanations and tasks should address students directly. They should be short and clear in order to make them easier to understand for students (see *Personalization Principle*).

Small number of questions: The number of tasks on a dynamic worksheet should be limited in order not to overwhelm students and prevent scrolling (see *Contiguity Principle*). If teachers want to include more questions they should consider creating a series of dynamic worksheets which could be connected using hyperlinks.

Use specific questions: General questions should be avoided in order to facilitate reaching the objective of the worksheet and guide students more effectively through mathematical discoveries (see ‘*guided discovery learning*’, [Bruner, 1961]). Additionally students should take notes on paper while working with dynamic worksheets in order to keep track of their explorations and findings.

The audience are learners: Information that is not relevant for a specific dynamic worksheet (e.g. general information, explanation of mathematical concepts) should be provided in a separate document in order to prevent distraction or confusion among students (see *Coherence Principle*).

Demonstration figure: If a dynamic worksheet is created for presentation purposes only, it might be better not to include explanations or tasks at all (see *Coherence Principle*), but to provide an additional document for other teachers who might want to use the dynamic worksheet for visualization purposes (e.g. lesson plan).

Author’s Guide and Evaluation Rubric for Dynamic Worksheets

In order to familiarize teachers with these design guidelines for dynamic worksheets, two additional workshop documents will be created. On the one hand, a so called *Author’s Guide for Dynamic Worksheets* will be designed which summarizes the design guidelines and gives additional information about their implementation. On the other hand, an *Evaluation Rubric for Dynamic Worksheets* will be created that allows teachers to assess the quality of dynamic worksheets from the GeoGebraWiki and to decide if modifications

according to the guidelines will be necessary in order to foster more effective student learning. Both these documents will be available online⁶.

8.3 Professional Development with GeoGebra

Feedback and data from the GeoGebra website confirm that most teachers who are currently using GeoGebra didn't receive any formal introduction or training in form of technology workshops or other professional development events. Many teachers started to use the software due to individual enthusiasm or encouragement by their colleagues. Apart from clusters of workshop activities in different countries no organized GeoGebra-related professional development events for in-service mathematics teachers could be traced yet. Although there are several universities and school districts that started to integrate GeoGebra into their teacher training programs, there is little collaboration, coordination, or communication among these sites [Hohenwarter and Lavicza, 2007, p. 50 – 51].

The International GeoGebra Institute was created in order to improve this situation and provide organized free professional development and instructional materials for mathematics teachers who require support with the use and integration of GeoGebra into their teaching environment.

8.3.1 The International GeoGebra Institute

The International GeoGebra Institute (IGI) represents an attempt to offer more structure and support for mathematics teachers who want to successfully integrate technology into their classrooms. On the one hand, it is intended to promote the teaching and learning of mathematics by providing the dynamic mathematics software GeoGebra for free and by offering free high-quality professional development in form of technology teacher training workshops that meet teachers' individual needs. On the other hand, the IGI will help to improve the design of GeoGebra based on the feedback participants give during and after the technology workshops as well as to coordinate research activities in relation to GeoGebra and its effective use for teaching and learning mathematics [Hohenwarter and Lavicza, 2007].

In order to intensify the training with GeoGebra and to encourage teachers to participate in follow-up workshops after they got introduced to the software for the first time, the International GeoGebra Institute will offer different levels of certification. According to their knowledge, skills, and creativity in terms of GeoGebra use and its effective integration into mathematics teaching and learning, teachers will be offered a series of GeoGebra workshops which focus on different aspects and difficulty levels of GeoGebra use in order to prepare teachers for new roles in the GeoGebra user community [Hohenwarter and Lavicza, 2007, p. 53].

⁶Author's Guide and Evaluation Rubric for Dynamic Worksheets:
www.geogebra.org/en/wiki/index.php/Workshop_materials

1. *GeoGebra Users* are teachers who attended basic GeoGebra workshops or learned how to use the software on their own. They are able to successfully use GeoGebra for their teaching and to integrate ready-to-use instructional materials like dynamic worksheets into their teaching practices.
2. *Creative GeoGebra Users* are teachers who not only know how to effectively use GeoGebra for their teaching, but also have the ability to create good-quality instructional materials with GeoGebra. They are willing to share these materials with the international user community on the GeoGebraWiki are able to give basic GeoGebra workshops in their own schools or districts, offering support for colleagues who want to successfully integrate GeoGebra into their teaching practices as well.
3. *GeoGebra Trainers* are teachers who “have the ability to carry out practitioner research on innovative practices” [Hohenwarter and Lavicza, 2007, p. 53] and present their results at the national level at conferences for mathematics teachers. They are able to offer GeoGebra workshops for teachers who want to reach the two certification levels described above.
4. *GeoGebra Institute Trainers* are “highly experienced GeoGebra presenters, trainers, and researchers” [Hohenwarter and Lavicza, 2007, p. 53] who are able to maintain the quality standards of professional development with GeoGebra set by the International GeoGebra Institute. They provide support and training for the international GeoGebra user community and participate actively in the implementation of local goals of different IGI sites around the world.

During the next years, several IGI sites will be established in the United States as well as Europe in order to build an accommodating network for teachers and researchers who want to work on various aspects of GeoGebra.

Although IGI sites will have specific locations, the idea of working together on open-source software is more important than the actual location of IGI sites. [...] The overall goal of IGI is to develop a supportive environment and continuous communication among participants and sites. According to this philosophy every IGI site would adopt ideas and materials to serve their local needs. [Hohenwarter and Lavicza, 2007, p. 52]

The first site of the International GeoGebra Institute is currently being established at Florida Atlantic University (FAU), USA. This site will be the role model for future IGI sites in the US and Europe and will provide a basis for an international cooperation. On the one hand, this first IGI site will organize and coordinate GeoGebra introductory workshops at different places in the US in order to spread the word about the software. On the other hand, more effective instructional materials will be developed in order to provide support for workshop presenters as well as mathematics teachers who want to use GeoGebra in their classrooms and effectively integrate it into their teaching practices.

Later on, other IGI sites will have unrestricted access to the newly developed GeoGebra documentation allowing them to base their activities on materials and experiences from the FAU-IGI site in Florida. Every IGI site will be free to adapt these materials to their local language and needs of the local teacher community by focusing on specific elements or even developing entirely new activities and workshop materials [Hohenwarter and Lavicza, 2007, p. 52]. For this purpose, content and structure of these materials need to be flexible in order to serve as many different IGI sites as possible.

8.3.2 Objectives of the International GeoGebra Institute

The structure and design of the International GeoGebra Institute are based on the following four main objectives [Hohenwarter and Lavicza, 2007]:

Providing free software that can be used by teachers and students in schools as well as at home. Since licence fees for commercial software often are expensive, they potentially prevent software from being used for teaching and learning mathematics [Suzuki, 2006, p. 26]. Although a lot of schools buy general licenses for certain mathematics software packages, teachers and students are usually not allowed to use the software at home as well. Due to this restricted use of the software, a lot of teachers don't want to integrate it into their teaching and spend the time to introduce it to their students.

In order to prevent this impediment, the IGI will provide technology professional development based on the dynamic mathematics software GeoGebra. Since the software is open source it can be downloaded from the Internet for free both in school and at home. Thus, the software is easier to distribute and its use is not restricted to certain people, computers, or locations. For this reason, teachers can be convinced more easily to consider using the software for teaching mathematics and introducing it to their students as well, allowing them to benefit from the potentiality of GeoGebra for learning and understanding mathematics.

Quality of professional development is often insufficient causing dissatisfaction among participating teachers. Younie [Younie, 2006, p. 394] reports that the provided professional development is "disorganised, lacking focus and too fragmented and text based". He also states that although professional development often increases teachers' confidence with computers, it rarely contributes "to the pedagogic expertise to help them make the most effective use of ICT in their lessons" [Office for Standards in Education (Ofsted), 2001] causing their use of technology for teaching to be largely unproductive and inconsistent.

In order to tackle this impediment, another objective of the International GeoGebra Institute is to provide high-quality professional development that (a) reduces the difficulties and impediments teachers have to face when being introduced to educational mathematics software, and that (b) fosters a successful integration of technology into teaching and learning mathematics. Based upon the study described in this thesis

new introductory workshops and accompanying instructional materials are being designed that deal with the basic use of the software and potentially prevent most common impediments that arise during the introduction process. Additionally, these materials are intended to prepare teachers for a more effective integration of GeoGebra into mathematics classrooms by providing them with best practice examples and introducing them to mathematical explorations, discovery learning, as well as the creation of own instructional materials with GeoGebra.

In order to increase the number of contact hours with the teachers, follow-up workshops that cover potential applications of GeoGebra for middle and high school level mathematics will be provided in addition to a full-day GeoGebra introductory workshop. In addition to thorough instruction and hands-on activities, teachers will have the opportunity to pick practice activities that meet their needs and make the workshop content more relevant for their individual situation. In all workshops, free supportive documentation handouts will be offered that contain detailed technical information, additional hands-on activities, ready-to-use materials, as well as tips and tricks concerning the use of GeoGebra and its successful integration into mathematics classrooms.

Ongoing support for teachers is often missing which makes it more challenging for teachers to effectively integrate the newly introduced technology into their everyday teaching. Although many teachers learn the basic operation of educational mathematics software in corresponding technology workshops, they often are not able to transfer their knowledge and skills into their classrooms and to use the software effectively for teaching. On the one hand, many teachers simply don't have enough confidence using the software after they got introduced to it in a professional development event. On the other hand, they don't know who to ask if questions and difficulties arise that are related to the technology and to getting support with its integration into their classroom practices.

An important component of the International GeoGebra Institute is to provide ongoing support for teachers. Apart from offering introductory as well as follow-up application workshops that can be adapted to the special needs of participants, another objective is to build a user community who share their knowledge about the use of GeoGebra and its effective integration into everyday teaching of mathematics. In order to provide support and help with occurring difficulties, the GeoGebra user forum⁷ was established in different languages. This forum is mainly maintained by an international community of teachers who use GeoGebra and enables teachers from all around the world to pose and answer questions related to GeoGebra.

Additionally, free instructional materials such as best practice examples, lesson plans, and ready-to-use interactive dynamic worksheets are provided on the Internet in order to facilitate the integration process of GeoGebra into classrooms and help teachers to effectively use the software with their students. For this purpose, the

⁷GeoGebra User Forum: www.geogebra.org/forum

GeoGebraWiki⁸ was established which provides pools of instructional materials in different languages that meet the local needs of mathematics teachers from different countries. By contributing to the GeoGebraWiki and providing their own dynamic worksheets and other instructional materials online, teachers support each other and help to improve mathematics teaching and learning by sharing their experiences of successful integration of technology into mathematics classrooms.

In order to support the international exchange of teaching materials, the IGI provides free web space for teachers to upload their GeoGebra files as well as accompanying materials and make them accessible to other teachers. All materials that are uploaded to the GeoGebra Upload Manager⁹ and the GeoGebra webserver are subject to the *Creative Commons License* allowing other teachers to copy, distribute, display, and use the materials for non-commercial purposes, as well as to create derivative materials under certain conditions¹⁰. This means, that teachers are not only allowed to use the dynamic worksheets and other instructional materials with their students, but also to modify and adapt them to their mathematical objectives and their students' individual needs.

Conducting research about GeoGebra will also be an important component of the International GeoGebra Institute. By getting researchers and teachers from all over the world involved with this project, research activities around GeoGebra and its effective use for teaching and learning mathematics will be organized in order to . . .

- analyze successful ways of introducing GeoGebra to an international user community of mathematics teachers in different types of schools and grade levels.
- develop high-quality professional development for mathematics teachers that fosters an effective integration of GeoGebra into mathematics classrooms.
- identify more effective methods of integrating GeoGebra successfully into teaching and learning of mathematics.
- develop interactive instructional materials with GeoGebra that foster active student learning as well as mathematical explorations and discoveries.
- assess the potential benefits of GeoGebra for learning and understanding of mathematics and for fostering better student achievement.
- adapt GeoGebra to the actual needs of teachers and students in different grade levels in order to make it easier to use for mathematics teaching and learning.
- further improve the usability of GeoGebra and nurture its further development.

⁸GeoGebraWiki: www.geogebra.org/en/wiki/index.php/Main_Page

⁹GeoGebra Upload Manager: www.geogebra.org/en/upload

¹⁰Creative Commons License: www.geogebra.org/en/cc_license/cc_license.htm

8.3.3 Design of Future GeoGebra Documentation

GeoGebra Application Workshops

GeoGebra application workshops will be designed as follow-up events for the GeoGebra introductory workshop (see section 8.2). In order to increase their flexibility concerning content and design a pool of mathematical content topics will be provided. Several content topics can be selected in order to create customized application workshops that either focus on the individual needs of the participating teachers (e.g. middle school, high school) or on certain mathematical topics (e.g. triangles, functions).

Each mathematical content topic will cover between 30 and 60 minutes of instruction and will provide additional practice activities for the participants. Similar documentation will be provided as for the introductory workshop (e.g. presenter's guide, handouts) and all materials will be available online as well.

The GeoGebra application workshops are intended to serve an international user community that is working with a variety of different mathematics curricula. A flexible workshop design needs to be implemented in order to meet the needs of mathematics teachers from all around the world. The 'patchwork-design' described increases the adaptability of workshops and documentation materials in terms of language, mathematical contents, and individual needs of international teacher communities. Since each application workshop can be composed of a selection of different mathematical topics a variety of follow-up workshops will be offered to teachers increasing the number of contact hours and intensifying the GeoGebra training in professional development events.

Since the documentation for the GeoGebra application workshops is going to be modified continuously and the number of content topics will be increased during the next months, all materials will be provided online¹¹.

Online Introductory Course

In order to provide GeoGebra training to even more teachers and overcome geographical as well as time limitations that prevent teachers from participating in professional development with GeoGebra, online courses will be offered. Apart from a basic introductory course for GeoGebra which will be based on to the *GeoGebra Introductory Workshop Guide* (see section 8.2.1), special content courses that meet the individual needs of different groups of teachers (e.g. geometry, algebra, calculus) will be provided as well.

In order to find out about difficulties and impediments that occur in online learning environments and potentially make it more difficult to learn how to use GeoGebra, further research needs to be conducted that focuses on the additional technical problems that arise in an online learning environment (e.g. how to use the corresponding content management system, how to communicate with the instructor and colleagues, how to submit exercises and homework). Additionally, the design of hands-on activities as well as theoretical input

¹¹GeoGebra application workshops: www.geogebra.org/en/wiki/index.php/Workshop_materials

needs to be analyzed in order to foster the learning progress of participating teachers and make the first contact with the software as easy as possible for them.

Introductory Book for GeoGebra

In addition to GeoGebra workshops and online courses, an introductory book for GeoGebra will be written. It will cover the basic use and functionality of the software and is intended to foster a successful integration of GeoGebra into everyday teaching by providing best practice examples and ready-to-use materials. All GeoGebra files and instructional materials described in the book will be provided online¹².

The introductory book for GeoGebra is intended to be a compendium for using GeoGebra for mathematics teaching as well as to support self-dependent exploration of the software for teachers who want to learn its basic use on their own without participating in a workshop or online course. Detailed instructions as well as tips and tricks concerning the operation of the software and its use for teaching will be included. Again, findings of the study described in this thesis as well as other research outcomes will be implemented in the design and structure of this book in order to prevent most common impediments and facilitate the first contact of teachers with GeoGebra.

Although the GeoGebra introductory book will be initially written in German and English, it will probably be translated to other languages in order to serve the international user community of GeoGebra. During the next few years, the content and usability of the introductory book need to be assessed in order to further improve and adapt it to teachers' needs.

Booklets for Mathematical Content Topics

Since the GeoGebra introductory book mentioned above will only provide basic information about the use of GeoGebra (like the GeoGebra Introductory Workshop Guide), special booklets covering a variety of different mathematical topics will be created as well. Like the GeoGebra application workshop topics, these booklets will focus on successfully integrating GeoGebra into teaching and learning mathematics in different grade levels and will contain best practice examples as well as ready-to-use instructional materials. Again, all materials will be available online¹³.

The GeoGebra booklets for mathematical content topics represent an attempt to summarize and organize materials and ideas related to a successful use of GeoGebra in mathematics classrooms. By making experiences and well-designed best practice examples available to other teachers, the international community of GeoGebra users will be strengthened and successful integration of the software into teaching and learning environments will be fostered. Again, the booklets will be offered for translation and adaptation to other languages and local needs of mathematics teachers and will be continuously enhanced according to suggestions of teachers who actually implemented the contents in their classrooms.

¹²GeoGebra Introductory Book: www.geogebra.org/en/wiki/index.php/Workshop_materials

¹³GeoGebra Booklets for Mathematical Content Topics: www.geogebra.org/en/wiki/index.php/Workshop_materials

Introductory Materials for Students

Once teachers start to effectively integrate GeoGebra into their classrooms instructional materials that facilitate a successful introduction of the software to students will be needed. On the one hand, these materials are intended to support teachers who want to invest the time and effort of teaching their students the basic use of GeoGebra. By allowing for self-dependent use of the software students can benefit from its potential as a general tool that facilitates learning and understanding of mathematics in many different ways. On the other hand, prepared introductory materials will facilitate students' first contact with GeoGebra by implementing the experiences of various workshop presenters as well as knowledge about the most common problems and impediments that arise during the introductory process of dynamic mathematics software.

The *GeoGebra Introductory Workshop Guide* as well as the content topics of the GeoGebra application workshops will serve as a basis for the development of introductory materials that are appropriate for students of different grade levels and mathematical content knowledge. Although independent use of GeoGebra and a good overview of its potential and application possibilities should be the long-term-goal for students and teachers, a different approach needs to be implemented in order to guarantee usability and an easy integration of these materials into 'traditional' teaching and learning environments. Instead of introducing students to all tools and features of the software at once, small portions of technical information could be embedded into mathematical content topics that are appropriate for students of certain grade levels. Materials should focus on the mathematical content instead of software use, which makes it easier for teachers to justify the amount of time spent teaching the use of GeoGebra and which helps to introduce the software as a versatile tool for dealing with different mathematical topics.

Providing introductory materials for students in combination with lesson plans and suggestions for teachers on how to effectively use these materials in classrooms could potentially foster a more successful introduction of the software to students and allows them to tap its full potential for learning and understanding of mathematical concepts. Nevertheless, research on the effectiveness and usefulness of such materials in teaching environments needs to be conducted in order to allow for an ongoing improvement of the materials as well as an optimization of benefits for students.

8.4 Conclusion and Further Research

The process of successfully integrating technology into mathematics teaching and learning progresses slowly and turned out to be rather complex. Today many teachers and students have access to computers and although appropriate software is available both in schools and at home, technology is rarely integrated substantially into everyday teaching. Being aware of the vital role teachers play in a technology-supported mathematics classroom, professional development opportunities need to be adapted in order to better prepare

teachers for this new challenge of effectively integrating technology into their teaching practice.

The research study conducted in the context of this dissertation represents a first step towards the goal of providing more successful introductory materials for professional development with dynamic mathematics software by identifying impediments teachers face when being introduced to this new technological tool. For different reasons, like its open-source nature and versatility, the dynamic mathematics software GeoGebra was selected from the pool of available educational mathematics software in order to evaluate a series of introductory technology workshops and assess the usability of the software itself. Based on the findings of this study, complexity criteria for dynamic geometry tools were defined in order to determine the general difficulty level of GeoGebra's construction tools. As shown in this thesis, the complexity criteria are also applicable to the tools of other dynamic geometry software packages. They could be useful for redesigning the 'traditional' introduction process of dynamic geometry tools and adapting the way in which they are presented to novices. New instructional materials could be developed or existing instructional materials for professional development could be modified with the goal of making the introduction of dynamic geometry software easier for mathematics teachers.

In the case of the dynamic mathematics software GeoGebra, this approach is currently being implemented in the design of a new introductory workshop and accompanying instructional materials for presenters as well as workshop participants. By preventing frequently occurring difficulties in terms of software use and design of workshop activities as well as incorporating characteristics of high-quality professional development, the first contact of mathematics teachers with GeoGebra is expected to be easier and more effective than in previous technology workshops. In order to set a high quality standard for professional development events with GeoGebra, follow-up application workshops as well as additional documentation, like online-courses, a GeoGebra introductory book, and instructional materials about GeoGebra and its integration into mathematics classrooms should also be developed in the future.

Based on the outcomes of the evaluation of introductory workshops and the usability of GeoGebra, further research studies need to be conducted in order to . . .

- evaluate the newly designed GeoGebra introductory workshop in terms of usability, effectiveness, and relevance for teachers,
- assess the quality of GeoGebra documentation, instructional materials, as well as professional development with GeoGebra which soon will be offered by the International GeoGebra Institute,
- gather data about the usability of GeoGebra that provides a basis for further development of the software and increases its usability for both teachers and students,
- assess the level of GeoGebra's effective integration into mathematics classrooms as well as its potential impact on instructional methods, classroom settings, instructional materials, and mathematical content taught, and

- assess the potential influence of GeoGebra on students' learning and understanding of mathematical concepts.

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Appendix A

GeoGebra Introductory Workshops

A.1 WS I: Basic Geometric Constructions

This workshop was given three times to a total of 42 participating secondary school mathematics teachers. It consisted of a short presentation of general information about the dynamic mathematics software GeoGebra, as well as four tasks focusing on basic geometric constructions and regular polygons.

Introduction

Time frame: 10 min

Features introduced: User interface, general use of GeoGebra

During the introduction, general information about the development and potential of GeoGebra for teaching and learning mathematics was presented. Additionally, the user interface and general use of the software were explained, and the participants were told where and how to get additional support in terms of software use and possible applications for teaching and learning mathematics (e.g. GeoGebra User Forum, online help document).

Activity 1: Line Bisector on Paper

Time frame: 10 min

Materials used: Paper, pencil, straightedge, compass

The first task was to construct a line bisector on paper using pencil, straight edge, and compass. After distributing the materials, the participants had time to try out this construction on their own. Then, the presenter clarified the construction steps by actually doing the construction and projecting it on screen:

1. Segment \overline{AB}

2. Circle with center A through point B
3. Circle with center B through point A
4. Line through intersection points of both circles

This activity was meant to clarify the construction steps in order to prepare the participants for the first use of GeoGebra.

Activity 2: Line Bisector with GeoGebra

Time frame: 10 min

Tools introduced: Segment between two points, Circle with center through point, Intersect two objects, Line through two points, Move, Move drawing pad, Zoom in / Zoom out

Features introduced: Construction protocol, Navigation bar

For the second activity the participants repeated the construction of a line bisector with GeoGebra which required the following construction steps (see figure A.1(a)).

1. Segment $a = \overline{AB}$ between points A and B
2. Circle c with center A through point B
3. Circle d with center B through point A
4. Intersect two circles c and d to get intersection points C and D .

Hint: Successively click on both circles in order to get both intersection points at once.

Hint: Zoom out of the construction if necessary in order to see both intersection points of the circles or move the drawing pad to adjust the visible area.

5. Line b through intersection points C and D is the linear bisector of segment \overline{AB}

Seven dynamic geometry tools as well as the features *Construction protocol* and *Navigation bar for construction steps* were introduced in this activity. Time was allocated for redoing the construction step-by-step and reviewing the construction process. After completing the construction, the drag test was applied to check the robustness of the construction.

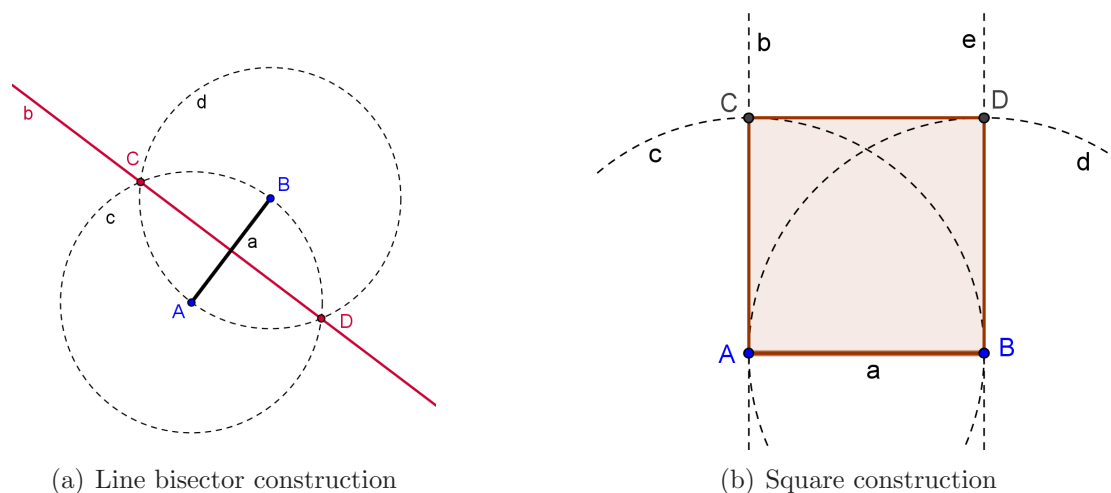


Figure A.1: Activities 2 and 3 of workshop I

Activity 3: Square

Time frame: 15 min

Tools introduced: Polygon, Perpendicular line, Show / hide object

Other tools used: Segment through two points, Circle with center through point, Intersect two objects, Move

In the third activity, participants constructed a square over a given segment by implementing the following construction steps (see figure A.1(b)).

1. Segment $a = \overline{AB}$ between points A and B
2. Circle c with center A through point B
3. Circle d with center B through point A
4. Perpendicular line b to segment \overline{AB} through point A
5. Perpendicular line e to segment \overline{AB} through point B
6. Intersect perpendicular line b with circle c to get intersection point C
7. Intersect perpendicular line e with circle d to get intersection point D
8. Polygon $ABCD$
9. Hint: You can hide the circle and perpendicular lines in order to ‘tidy up’ the construction.

In total, seven tools were used in this activity and the tools *Perpendicular line*, *Polygon*, and *Show / hide object* were introduced for the first time. After finishing the construction, the drag test was applied in order to check whether or not the square was constructed correctly.

Activity 4: Circumscribed Circle of a Triangle

Time frame: 15 min

Tools introduced: Line bisector

Other tools used: Polygon, Intersect two objects, Circle with center through point, Move

Features introduced: Rename

In the last activity of this workshop, participants constructed the circumscribed circle of a triangle by implementing the following construction steps (see figure A.2(a)).

1. Polygon *poly1* through points A , B , and C
2. Line bisectors d , e , and f
3. Intersection point D of two line bisectors

Hint: Successively click on two of the line bisectors in order to intersect them.

4. Circle with center D through one of the vertices of triangle ABC
5. Rename center of circumscribed circle

This activity was meant to introduce the tool *Line bisector*, which is one of five tools used in this construction. After finishing the construction and renaming the intersection point, the drag test was applied in order to check the robustness of the construction as well as to examine the positions of the circumcenter for different types of triangles (e.g. right triangle).

Home Exercise 1: Equilateral Triangle

Tools intended to use: Segment through two points, Circle with center through point, Intersect two objects, Polygon, Show / hide object, Move

For home exercise the participants had to construct an equilateral triangle using some of the tools introduced in workshop I (see figure A.2(b)).

One possible solution for the construction of an equilateral triangle with GeoGebra uses the following construction steps. This construction only requires the use of tools that were already introduced during workshop I and was therefore thought to be appropriate for the skills of the participants. Additionally, participants were supposed to apply the drag test in order to check the robustness of their construction.

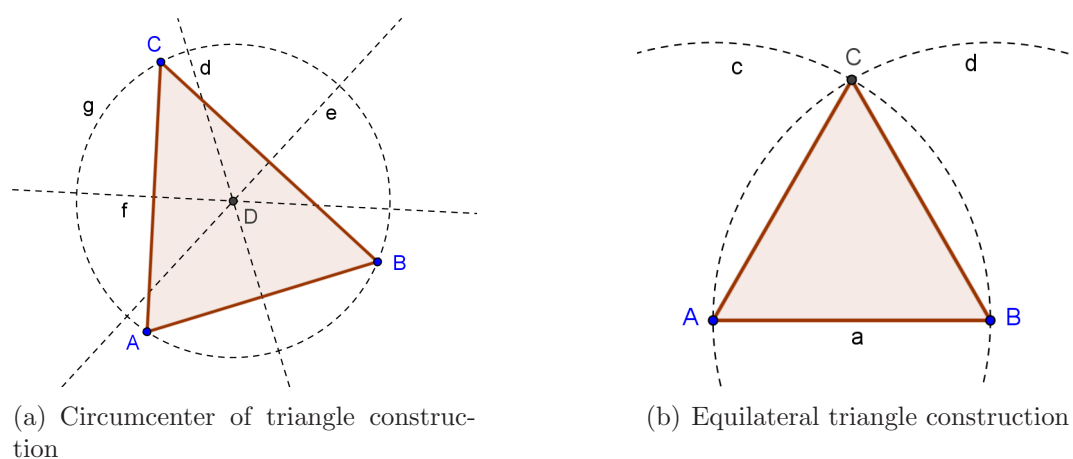


Figure A.2: Activity 4 and home exercise 1 of workshop I

1. Segment $a = \overline{AB}$ between points A and B
2. Circle c with center A through point B
3. Circle d with center B through point A
4. Intersect circles c and d to get intersection point C
5. Polygon ABC
6. Hint: You can hide the circles in order to ‘tidy up’ your construction.

A.2 WS II: Angles, Transformations, and Inserting Images

Workshop II was given three times to a total of 43 participants. It consisted of four activities that focused on the introduction of angles and transformations. Additionally, participants learned how to insert an image into the graphics window of GeoGebra and how to use it to enhance a dynamic figure.

Homework Exercise Discussion

Time frame: 5 min

This workshop started with the presentation of the previous day’s home exercise which was given by either a volunteering participant or the workshop presenter. The construction process was discussed and different ways of solving this task were presented (e.g. creating an equilateral triangle by rotating a segment).

Activity 1: Parallelogram with Angles

Time frame: 15 min

Tools introduced: Parallel line, Angle

Other tools used: Segment between two points, Intersect two objects, Polygon, Show / hide object, Move

Features introduced: Grid, Point capturing, Context menu, Properties dialog

The first activity of this workshop was to construct a parallelogram and measure all its interior angles by applying the following construction steps (see figure A.3(a)).

1. Segment $a = \overline{AB}$ between points A and B
2. Segment $b = \overline{BC}$ between points B and C
3. Parallel line c to segment a through point C
4. Parallel line d to segment b through point A
5. Intersection point D of lines c and d
6. Polygon $ABCD$
7. Hide parallel lines
8. Interior angles of the parallelogram

A total of seven tools were used in this activity and the tools *Parallel line* and *Angle* were introduced for the first time. After finishing the construction, the drag test was applied in order to check the robustness and correctness of the parallelogram construction.

By means of this task four GeoGebra features were introduced. On the one hand, features *Grid* from the *View* menu and feature *Point capturing* from the *Options* menu were used to facilitate the creation of points with integer coordinates. On the other hand, the *Context menu* (right click on object, MacOS: command click) was introduced in order to open the *Properties dialog* which allows changes to the properties of objects used in a construction (e.g. color, line style).

Activity 2: Drawing Tool for Symmetric Figures

Time frame: 15 min

Tools introduced: New point, Mirror object at line

Other tools used: Line through two points, Segment between two points, Move

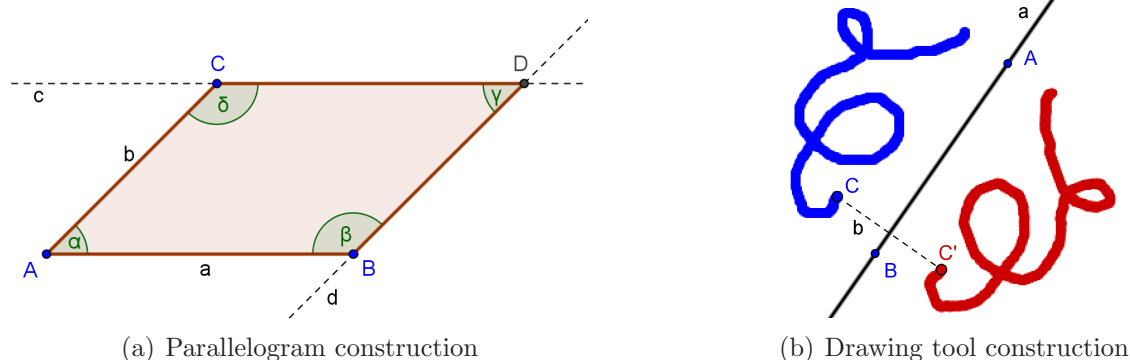


Figure A.3: Activities 1 and 2 of workshop II

Features introduced: Trace on

During the second activity, a tool to draw symmetric figures was created by applying the following construction steps (see figure A.3(b)).

1. Line a through points A and B
2. Point C
3. Mirror point C at line a to get point C'
4. Segment b between points C and C'
5. Feature *Trace on* for points C and C'
6. Task: Move point C in order to draw a symmetric figure.

A total of five tools were used in this activity and the tools *New point* and *Mirror object at line* were introduced for the first time. Additionally, the *Trace on* feature was introduced in order to keep track of the moveable point C and its mirrored image C' . Thus, the participants could experiment with drawing symmetric figures.

Activity 3: Inserting a Background Image

Time frame: 5 min

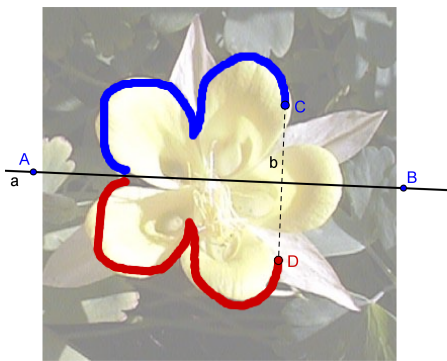
Tools introduced: Insert image

Other tools used: Move

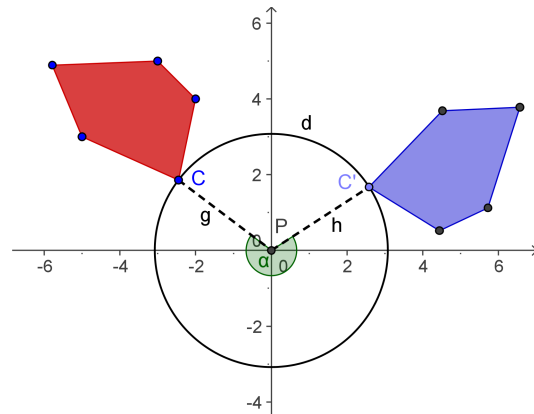
Features introduced: Background image

The third activity was supposed to enhance the construction of the drawing tool for symmetric figures by inserting a ‘symmetry picture’ into the background of the GeoGebra construction in order to explore symmetry axes of the image using the drawing tool (see figure A.4(a)). The new tool *Insert image* was introduced and properties of the image were changed using the *Properties dialog* of GeoGebra.

Due to technical problems, not all of the participants had Internet access during this workshop. Therefore, they didn’t have access to the provided picture and couldn’t work along with the presenter when he showed how to insert an image into a GeoGebra file, change its filling and set it as a background image. Instead, the participants were supposed to take notes in order to be able to redo the construction as a home exercise.



(a) Flower symmetry construction



(b) Rotation of polygon construction

Figure A.4: Activities 3 and 4 of workshop II

Activity 4: Rotation of a Polygon

Time frame: 20 min

Tools introduced: Rotate object around point by angle

Other tools used: Polygon, New point, Circle with center through point, Segment between two points, Angle, Move

The last activity of this workshop was to rotate a polygon around a point through a given angle by applying the following construction steps (see figure A.4(b)).

1. Polygon $ABC \dots$ with vertices A, B, C, \dots
2. New point P
3. Circle d with center P through one of the vertices of the polygon, e.g. C

4. Segment $g = \overline{CP}$ between points C and P
5. New point C' on circle d
Hint: Move point C' in order to check if it is really on the circle.
6. Segment $h = \overline{PC'}$ between points P and C'
7. Angle α between points C , P , and C'
8. Rotate polygon around point P by angle α to get the image of the initial polygon
9. Task: Move point C' along the circle in order to rotate the polygon.

In this activity, seven different tools were used to create the construction and the *Rotate object around point by angle* tool was introduced for the first time.

Home Exercise 2: Drawing Tool to Check for Axes of Symmetry

Tools intended to use: Insert image, Line through two points, New point, Mirror object at line, Segment between two points, Move

As a home exercise the participants were supposed to redo the drawing tool for symmetric figures (see figure A.4(a)). They had to search the Internet for a picture of a symmetric object and insert it into their GeoGebra construction as a background image. Furthermore, they were supposed to change the properties of the image to make it more transparent in order to improve the visibility of the construction on top of the background image. A possible solution for this home exercise could be created by implementing the following instructions.

1. Task: Search for a suitable image on the Internet and save it on your computer.
2. Insert the image into GeoGebra
3. Change properties of the picture (background image, filling)
4. Line a through points A and B
5. Point C
6. Mirror point C at line a to get point C'
7. Segment b between points C and C'
8. Feature *Trace on* for points C and C'
9. Task: Move point C with the mouse in order to check the picture for axes of symmetry.

Hint: You might have to adjust the position of the line of reflection.

In this construction version, only familiar tools had to be used, and each construction step had already been presented at some point during workshop II.

A.3 WS III: Coordinates and Equations

Introductory workshop III was given three times to a total of 44 participants. It provided four new activities focusing on the introduction of coordinates and equations in GeoGebra.

Home Exercise Discussion

Time frame: 5 min

Workshop III started with the discussion of the last home exercise. Problems related to searching for appropriate pictures on the Internet and downloading them were also addressed by the participants.

Activity 1: Coordinates of Points

Time frame: 15 min

Tools used: Mirror object at line, Move

Algebraic input introduced: Coordinates of point, extract coordinates

Features introduced: Algebra window, free and dependant objects, coordinate axes, grid, labels of objects

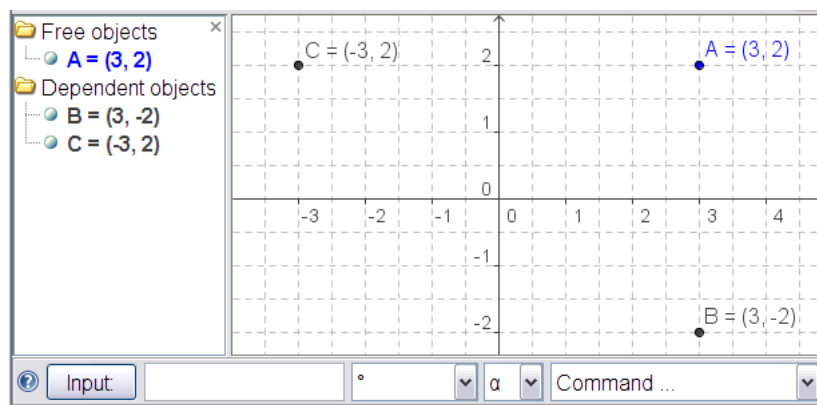


Figure A.5: Coordinates of points

The first activity of this workshop was used to introduce the algebra window and input field of GeoGebra. The participants learned how to create a new point by using the keyboard, as well as to extract its x - and y -coordinate. Then, they mirrored the point at both coordinate axes to examine how this would impact the coordinates (see figure A.5). Thereby, the following instructions were implemented.

1. Open algebra window and show axes
2. Create new point $A = (3, 2)$
3. Change displayed label of point A to show name and value
4. Task: Move point A with the mouse to change its coordinates and try to change them in the algebra window as well.
5. Get x -coordinate of point A : `xCoordinate = x(A)`
6. Get y -coordinate of point A : `yCoordinate = y(A)`
7. Mirror point A at the y -axis
8. Mirror point A at the x -axis
9. Task: Move point A to another position and make a conjecture about the coordinates of the mirrored points.

This task was focused on the input and modification of points using the *input field* and *algebra window* of GeoGebra. The features *Axes* and *Grid* were introduced and differences between *free* and *dependent objects* shown in the algebra window were discussed. Additionally, different kinds of *labels* were introduced (e.g. name & value).

Activity 2: Linear Equation

Time frame: 15 min

Tools introduced: Slider, Slope

Other tools used: Move

Algebraic input introduced: Line equation

Features introduced: Redefine

In the second activity of this workshop, participants learned how to enter linear equations in slope intercept form using the input field (see figure A.6). The following instructions were implemented.

1. Create line: `line: y = 0.8 x + 3.2`
2. Task: Modify the parameters of the line in the algebra window and try to move the line with the mouse as well.
3. Create sliders a and b
4. Rename slider a to m

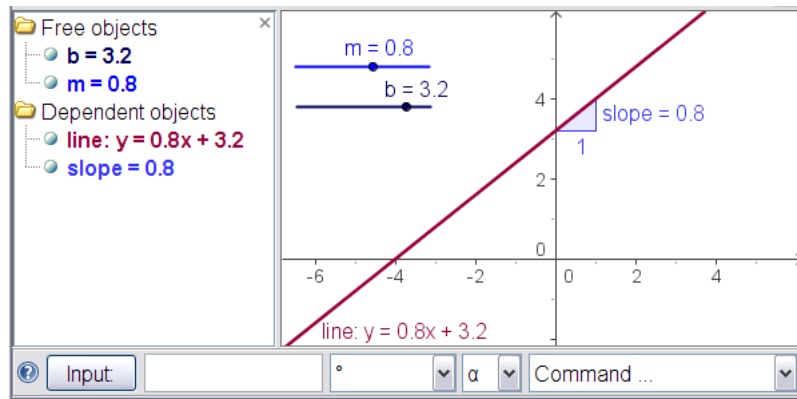


Figure A.6: Linear equation

5. Redefine line: `line: y = m x + b`

Hint: Use * or space to indicate multiplication

6. Create slope of the line

In this activity tools *Slider* and *Slope*, as well as the input of a linear equation were introduced. Additionally, the feature *Rename* was applied for the first time, allowing the use of belatedly created sliders to control the parameters of the linear equation.

Activity 3: Slope Triangle

Time frame: 20 min

Tools introduced: Insert text

Other tools used: Move

Commands introduced: Slope

Algebraic input introduced: Calculations

Features introduced: Static text, dynamic text, auxiliary objects

In the third activity, participants manually constructed the slope triangle of a line without using the *Slope* tool (see figure A.7). The following instructions were implemented.

1. Line a through points A and B
2. Perpendicular line b to x -axis through point A
3. Perpendicular line c to y -axis through point B

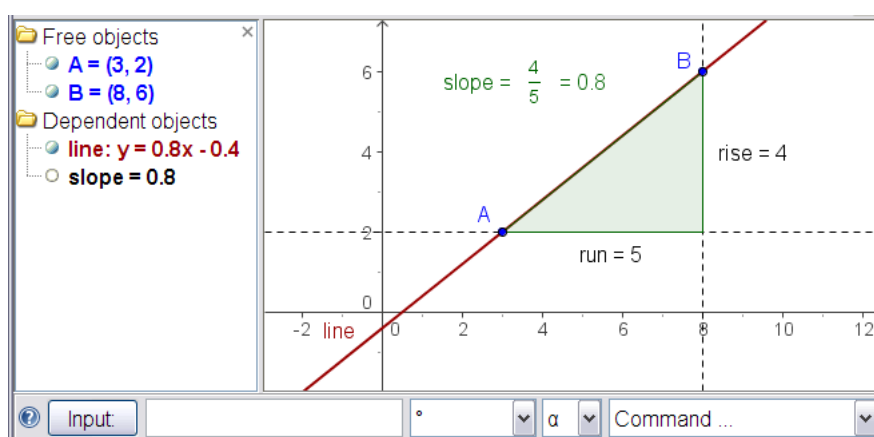


Figure A.7: Slope triangle construction

4. Intersection point C of perpendicular lines b and c
5. Polygon ABC
6. Calculate rise: $\text{rise} = y(B) - y(A)$
7. Calculate run: $\text{run} = x(B) - x(A)$
8. Insert dynamic text: "rise = " + rise
9. Insert dynamic text: "run = " + run
10. Calculate slope of the initial line: $\text{slope} = \text{Slope}[a]$
11. Insert dynamic text: "slope = rise / run = " + slope

In this activity, six tools were used and the *Insert text* tool was introduced for the first time in order to create static and dynamic text within the graphics window. Additionally, the input of calculations as well as the new command *Slope* were used within the construction process.

Activity 4: Parabola

Time frame: 5 min

Tools used: Move

Commands introduced: Vertex

Algebraic input introduced: Parabola equation

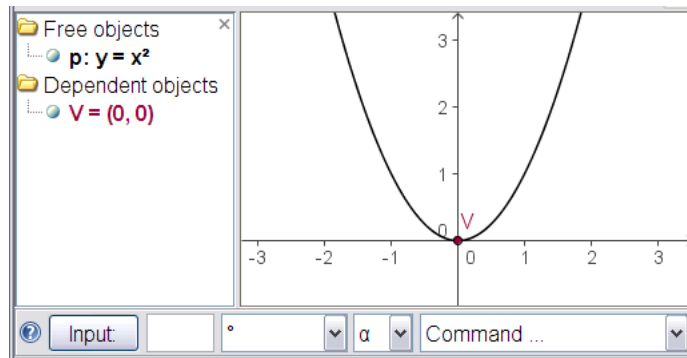


Figure A.8: Parabola

In the last activity of this workshop, participants learned how to enter a parabola using the input field and apply the new command *Vertex* to it (see figure A.8). The following instructions were implemented.

1. Enter parabola: $p: y = x^2$
2. Task: Move parabola p with the mouse and try to change its equation in the algebra window as well.
3. Vertex V of parabola p : $V = \text{Vertex}[p]$
4. Task: Find out about the connection between coordinates of vertex V and parameters of the equation of parabola p .

The command *Vertex* was the second command introduced to the participants. Since it can just be applied to a conic section and not to a function, the differences between entering a parabola or a quadratic function had to be explained by the instructor ($y = \dots$ versus $f(x) = \dots$).

Home Exercise 3: Quadratic Equation

Tools intended to use: Slider, Move

Algebraic input intended to use: Parabola

Commands intended to use: Vertex

In home exercise 3 participants had to link sliders to the parameters of a quadratic equation representing a parabola. Using the feature *Trace on* they were supposed to experiment with the trace of the parabola's vertex and make a conjecture about its movements depending on the parameter values (see figure A.9). The following instructions represent one possible way of creating this GeoGebra construction which requires the use of several already introduced tools as well as some algebraic input and the *Vertex* command.

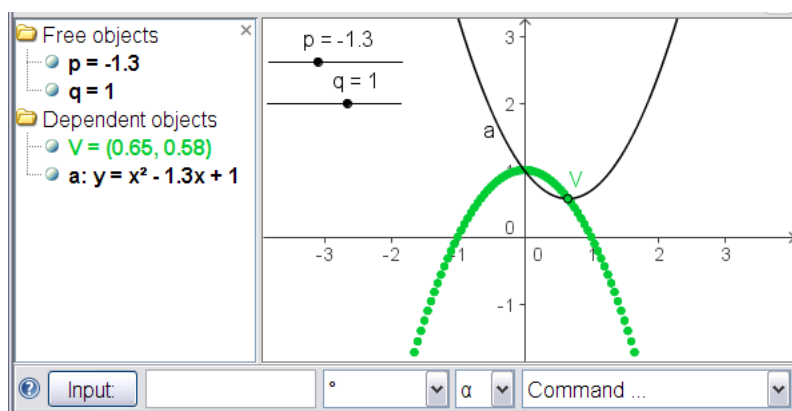


Figure A.9: Quadratic equation and trace of vertex

1. Create sliders p and q
2. Enter quadratic equation to get parabola a : $y = x^2 + p x + q$
Hint: Move sliders p and q to change the equation of the parabola.
3. Get vertex V of parabola a : $V = \text{Vertex}[a]$
4. Task: Move parabola a using the sliders and make a conjecture about the path of vertex V . Show the trace of vertex V and confirm the conjecture

A.4 WS IV: Functions and Export of Pictures

Workshop IV was given three times to a total of 44 participants. Four new activities were offered focusing on the introduction of functions and the export of static pictures in GeoGebra.

Home Exercise Discussion

Time frame: 5 min

At the beginning of the last introductory workshop, the third home exercise was discussed. The instructor focused on developing a conjecture about the movement of a parabola's vertex and then confirming it by activating its trace. Thus, the participants were introduced to one mode of experimental learning with GeoGebra.

Activity 1: Polynomial Functions

Time frame: 10 min

Tool used: Move

Commands introduced: Root, Extremum

Algebraic input introduced: Polynomial function

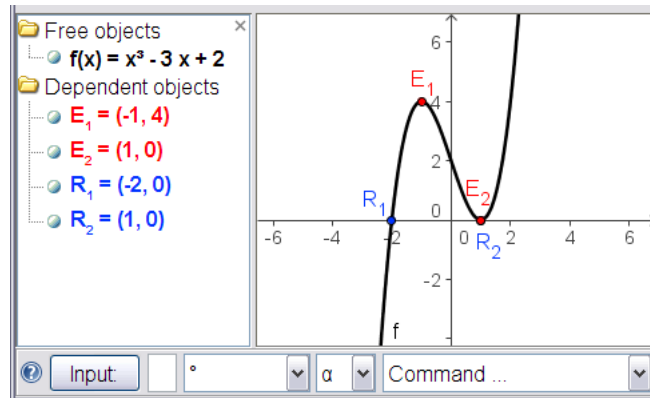


Figure A.10: Polynomial function

The first activity of this workshop was dedicated to polynomial functions. Participants learned how to enter a polynomial function in GeoGebra and how to display its roots and extrema using new commands (see figure A.10). The following instructions give an overview of this activity.

1. Enter cubic polynomial function f : $f(x) = x^3 - 3x + 2$
2. Task: Move polynomial f with the mouse and watch how its equation adapts automatically.
Hint: Change the parameters in the algebra window to restore the original function.
3. Roots of polynomial f : $R = \text{Root}[f]$
4. Extrema of polynomial f : $E = \text{Extremum}[f]$
5. Task: Move polynomial f with the mouse and watch the roots and extrema changing

Although both newly introduced commands can be applied to all polynomial functions, the instructor explained why they wouldn't work for other non-polynomial functions.

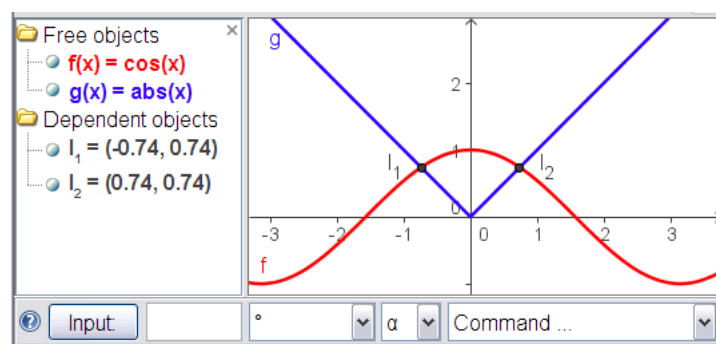


Figure A.11: Library of functions

Activity 2: Library of Functions

Time frame: 15 min

Tools used: Move, Intersect two objects

Algebraic input introduced: Functions

For the second activity, the GeoGebra ‘library of functions’ was introduced. On the one hand, the participants were taught how to input some of the pre-defined functions (e.g. trigonometric functions, absolute value function, logarithmical function). On the other hand, they also learned how to use the *Intersect two objects* tool in order to intersect two functions or to get a particular root by intersecting a function with the x -axis. Furthermore, the instructor showed how to solve a given equation graphically by entering both sides as functions and intersect their graphs in order to get the graphical solution of the equation (see figure A.11).

Type of Function	Input
Trigonometric functions	$\sin(x)$, $\cos(x)$, ...
Absolute value function	$\text{abs}(x)$
Logarithmic functions	$\lg(x)$, $\ln(x)$

Table A.1: Library of functions for activity 2 of WS IV

Since GeoGebra offers a selection of pre-defined functions in the form of a menu next to the input field, the participants were encouraged to try out some of them on their own. According to questions posed during this phase of the workshop, the instructor presented several examples of functions. This activity turned out to be slightly different in each of the three workshop sessions. Table A.1 gives an overview about the presented pre-defined functions.

Activity 3: Tangent and Slope Function

Time frame: 15 min

Tools introduced: Tangent

Other tools used: New point, Move

Algebraic input introduced: Point with special coordinates

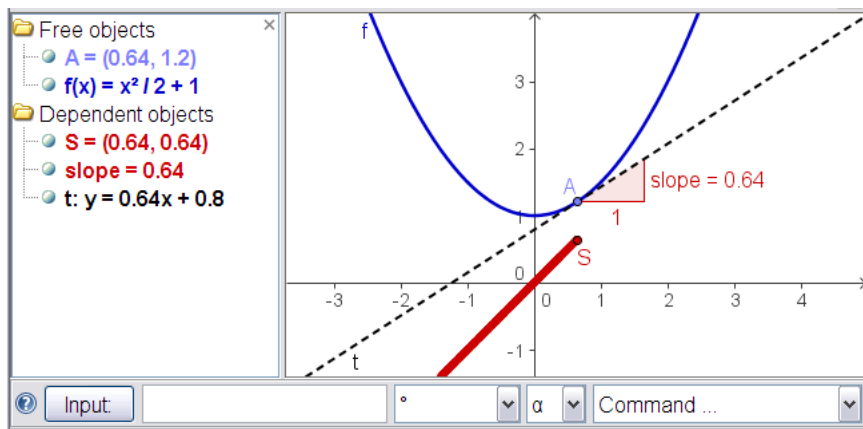


Figure A.12: Tangent and slope function

In the third activity, participants learned how to place a point on a function, create a tangent, and get the corresponding slope function as trace of a special point (see figure A.12). The following instructions were implemented.

1. Enter polynomial function f : $f(x) = x^2/2 + 1$
2. Create new point A on function f

Hint: Move point A along function f to check whether it is restricted to the function graph.
3. Create tangent t to function f through point A
4. Get the slope of tangent t : $\text{slope} = \text{Slope}[t]$
5. Define point S : $S = (x(A), \text{slope})$
6. Turn on the trace of point S
7. Move point A to create the slope function

In this activity, participants used the tool *New point* in order to place a point on a function. This allowed them to create the tangent through this point using the tool *Tangents* which was introduced in this activity. Additionally, the command *Slope* was used in order to define a point whose trace showed the slope of the function as dependent on the x -values of the moveable point.

Activity 4: Export of Static Pictures

Time frame: 15 min

Features introduced: Export drawing pad to clipboard, export drawing pad as picture

The last activity of this workshop was to export the GeoGebra drawing pad as a static picture and insert it into an MS Word document. First, participants learned how to export the drawing pad to the clipboard in order to insert it directly into a Word document.

1. Create a construction in GeoGebra.
2. Use the selection rectangle to determine which part of the drawing pad should be exported.
3. Use the *File* menu to export the selection to the clipboard (*Export – Drawing pad to clipboard*).
4. Open a new MS Word document.
5. Paste the picture using menu *Edit – Paste*.

Secondly, participants also learned how to export the drawing pad as a picture and save it in a folder before inserting it into an MS Word document.

1. Create a construction in GeoGebra.
2. Use the selection rectangle to determine which part of the drawing pad should be exported.
3. Use the *File* menu to export the selection to a file (*Export – Drawing pad as picture (png, eps)*).

Hint: Choose a scale factor and resolution before saving the picture as a file.

4. Open a new MS Word document.
5. Insert the picture using menu *Insert – Picture – From File ...* and selecting the corresponding picture file.

For this activity, the construction created in activity 3 was used in order to practice exporting static pictures. Additionally, advantages of both export possibilities were discussed.

Home Exercise 4: ‘Function Domino’ Game

As a home exercise, the participants were supposed to create a so called ‘Function Domino’ game in MS Word (see figure A.13). For this purpose, they had to create function graphs in GeoGebra and export them as static pictures which could be inserted into an MS Word document. The following instructions illustrate how to create one card for the ‘Function Domino’ game.

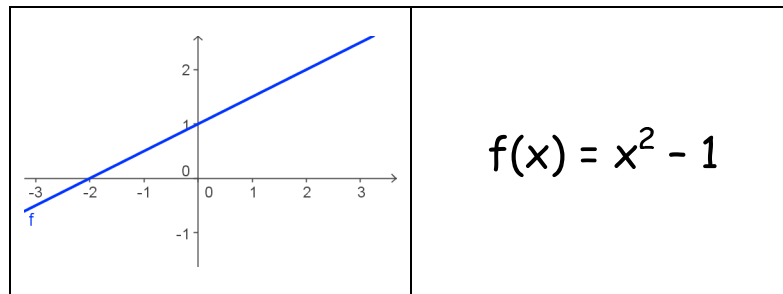


Figure A.13: ‘Function Domino’ card

1. Plot an arbitrary function in GeoGebra.
2. Select part of the drawing pad which is supposed to be visible in the picture.
3. Export the selection as a static picture.
4. Open a new MS Word document.
5. Create a table with one row and two columns in MS Word.
6. Insert the picture into the left cell of the table.

Hint: Resize the picture if necessary.
7. Enter the equation of another function into the right cell of the table.

This home exercise was discussed on the following day in the beginning of an additional GeoGebra workshop which was not part of the evaluation process.

Appendix B

Evaluation Instruments

Survey I: Computer Literacy

NSF MSP Summer Institute Evaluation (Survey I) Introduction to GeoGebra

Code:

Mom	Dad	Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith** Dad's name: **Dave Jones** YoB: **1976**
Code: **MSDJ1976**

General Information

1. male female 2. Age: _____ 3. Teaching years: _____

4. Which grade level(s) / courses are you teaching? Please check!

6 7 8 9 10
 Pre-Algebra Algebra Geometry
 Pre-Calculus Calculus Advanced

Technology Use Outside Classroom

1. How many days per week do you use a computer...
 ...at home: _____ ...at school (not during classes!): _____

2. I use my computer at... ...home ...school

	yes	no	yes	no
<input type="radio"/> to check e-mail	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to chat with friends	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to prepare lessons	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to create my own teaching materials	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to look for teaching materials from the Internet	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to find content information on the Internet	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to keep records of my student's grades	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> others: _____	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

3. If you use a computer to prepare your lessons:
 Which kind of technology do you use to prepare your lessons?

	yes	no
<input type="radio"/> Presentation software (e.g. PowerPoint)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Word processing software (e.g. Word)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Spreadsheets (e.g. Excel)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Dynamic Geometry Software (e.g. Geometer's Sketchpad)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Computer Algebra Systems (e.g. Derive)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Graphing calculators (e.g. TI 83)	<input type="checkbox"/>	<input type="checkbox"/>

NSF MSP Summer Institute Evaluation (Survey I)
Introduction to GeoGebra

Technology Use in Classroom

1. How many days per week do you use a computer for teaching? _____ days

2. How many times per week do your students actively use computers in class? _____ times

3. How do you use a computer during your class?

	yes	no
<input type="radio"/> as a presentation tool with a projector	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> for FCAT preparation	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> to help my students discover mathematical concepts	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> for educational games as a reward	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> others: _____	<input type="checkbox"/>	<input type="checkbox"/>

4. Which kind of technology do you use for teaching?

	yes	no
<input type="radio"/> Presentation software (e.g. PowerPoint)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Word processing software (e.g. Word)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Spreadsheets (e.g. Excel)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Dynamic Geometry Software (e.g. Geometer's Sketchpad)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Computer Algebra Systems (e.g. Derive)	<input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Graphing calculators (e.g. TI 83)	<input type="checkbox"/>	<input type="checkbox"/>

Computer Skills

- Do you know how to...
- | | yes | no |
|---|--------------------------|--------------------------|
| <input type="radio"/> Left click (MacOS: click) | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Right click (MacOS: Apple-click) | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Click and drag | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Create a new folder | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Select a file or folder | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Select a series of files or folders | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Save a file | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Delete a file or folder | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Copy and paste text | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Make a screenshot | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Insert pictures into a Word file | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Make printouts | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Open a web browser | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Find information on the Internet | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Upload files to the Internet | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Create a web page | <input type="checkbox"/> | <input type="checkbox"/> |
| <input type="radio"/> Use html code | <input type="checkbox"/> | <input type="checkbox"/> |

Thanks for your participation!

Survey II: GeoGebra Features

NSF MSP Summer Institute Evaluation (Survey II) Introduction to GeoGebra

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith** Dad's name: **Dave Jones** YoB: **1976**
Code: **MSDJ1976**

General Information...

... about the computer you used in the GeoGebra workshops this week:

	yes	no
1. Does the notebook computer you used this week belong to you?	<input type="checkbox"/>	<input type="checkbox"/>
2. Did you use the touchpad in order to operate GeoGebra?	<input type="checkbox"/>	<input type="checkbox"/>
3. Did you use a Mac Notebook?	<input type="checkbox"/>	<input type="checkbox"/>
4. Did you have major problems with your computer in general?	<input type="checkbox"/>	<input type="checkbox"/>

GeoGebra Features

Please rate whether the use of the following GeoGebra features was difficult.

(5 very difficult, 0 very easy)

<input type="radio"/> Construction protocol	5	4	3	2	1	0
<input type="radio"/> Navigation bar	5	4	3	2	1	0
<input type="radio"/> Rename objects	5	4	3	2	1	0
<input type="radio"/> Context menu	5	4	3	2	1	0
<input type="radio"/> Properties dialog	5	4	3	2	1	0
<input type="radio"/> Grid	5	4	3	2	1	0
<input type="radio"/> Point capturing	5	4	3	2	1	0
<input type="radio"/> Trace of an object	5	4	3	2	1	0
<input type="radio"/> Background image	5	4	3	2	1	0
<input type="radio"/> Labeling objects	5	4	3	2	1	0
<input type="radio"/> Redefine objects	5	4	3	2	1	0
<input type="radio"/> Auxiliary objects	5	4	3	2	1	0
<input type="radio"/> Insert static text	5	4	3	2	1	0
<input type="radio"/> Insert dynamic text	5	4	3	2	1	0
<input type="radio"/> Create a point on an object	5	4	3	2	1	0

Anything else you wanted to say?

Please write down what you liked / what you didn't like about these GeoGebra workshops. We are also looking forward to getting suggestions on how to improve GeoGebra! (Please use the backside if you need more space.)

Thanks for your participation!

Judith Preiner

Rating Survey for Workshop I

NSF MSP Summer Institute Evaluation Workshop 1: Segments, Circles, Triangles

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith** Dad's name: **Dave Jones** YoB: **1976**
Code: MSDJ1976












1. Please rate whether the GeoGebra workshop activities have been difficult.

(5 very difficult, 0 very easy)

- | | | | | | | |
|--|---|---|---|---|---|---|
| <input type="radio"/> Line bisector construction on paper | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Line bisector construction with GeoGebra | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Orthocenter of a triangle construction | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Square over a segment | 5 | 4 | 3 | 2 | 1 | 0 |

2. Please rate whether the following GeoGebra tools were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | |
|--|---|---|---|---|---|---|
|  Segment through two points | 5 | 4 | 3 | 2 | 1 | 0 |
|  Circle with center through point | 5 | 4 | 3 | 2 | 1 | 0 |
|  Intersect two objects | 5 | 4 | 3 | 2 | 1 | 0 |
|  Line through two points | 5 | 4 | 3 | 2 | 1 | 0 |
|  Move | 5 | 4 | 3 | 2 | 1 | 0 |
|  Polygon | 5 | 4 | 3 | 2 | 1 | 0 |
|  Line bisector | 5 | 4 | 3 | 2 | 1 | 0 |
|  Show / hide object | 5 | 4 | 3 | 2 | 1 | 0 |
|  Move drawing pad | 5 | 4 | 3 | 2 | 1 | 0 |
|  Zoom in / Zoom out | 5 | 4 | 3 | 2 | 1 | 0 |
|  Perpendicular line | 5 | 4 | 3 | 2 | 1 | 0 |

3. What did you like about this GeoGebra workshop?

4. Is there anything you didn't like about this GeoGebra workshop? If yes, please explain!

Thanks for your participation!

Rating Survey for Workshop II

NSF MSP Summer Institute Evaluation
Workshop 2: Angles and Transformations

Code:

Mom		Dad		Year of Birth		

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: Mary Smith Dad's name: Dave Jones YoB: 1976
 Code: MSDJ1976














1. Please rate whether the GeoGebra workshop activities have been difficult.

(5 very difficult, 0 very easy)

- | | | | | | | | |
|-----------------------|---------------------------------------|---|---|---|---|---|---|
| <input type="radio"/> | Parallelogram and Angles | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Symmetry Construction | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Background Image and Axes of Symmetry | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Rotation of a Polygon | 5 | 4 | 3 | 2 | 1 | 0 |

2. Please rate whether the following GeoGebra tools were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | | |
|---|----------------------------------|---|---|---|---|---|---|
|  | Segment through two points | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Circle with center through point | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Intersect two objects | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Line through two points | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Move | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Polygon | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Show / hide object | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Parallel line | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Angle | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Mirror at line | 5 | 4 | 3 | 2 | 1 | 0 |
|  | New point | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Rotate around point | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Insert image | 5 | 4 | 3 | 2 | 1 | 0 |

3. What did you like about this GeoGebra workshop?

4. Is there anything you didn't like about this GeoGebra workshop? If yes, please explain!

Thanks for your participation!

Rating Survey for Workshop III

NSF MSP Summer Institute Evaluation
Workshop 3: Coordinates and Equations

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith** Dad's name: **Dave Jones** YoB: **1976**
 Code: MSDJ1976












1. Please rate whether the GeoGebra workshop activities have been difficult.

(5 very difficult, 0 very easy)

- | | | | | | | |
|---|---|---|---|---|---|---|
| <input type="radio"/> Coordinates of Points | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Slope Intercept Form of a Linear Equation | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Slope Triangle | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Parabola | 5 | 4 | 3 | 2 | 1 | 0 |

2. Please rate whether the following GeoGebra tools were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | |
|---|---|---|---|---|---|---|
|  Intersect two objects | 5 | 4 | 3 | 2 | 1 | 0 |
|  Line through two points | 5 | 4 | 3 | 2 | 1 | 0 |
|  Move | 5 | 4 | 3 | 2 | 1 | 0 |
|  Polygon | 5 | 4 | 3 | 2 | 1 | 0 |
|  Show / hide object | 5 | 4 | 3 | 2 | 1 | 0 |
|  Perpendicular line | 5 | 4 | 3 | 2 | 1 | 0 |
|  Parallel line | 5 | 4 | 3 | 2 | 1 | 0 |
|  New point | 5 | 4 | 3 | 2 | 1 | 0 |
|  Slider | 5 | 4 | 3 | 2 | 1 | 0 |
|  Slope | 5 | 4 | 3 | 2 | 1 | 0 |
|  Insert Text | 5 | 4 | 3 | 2 | 1 | 0 |

3. Please rate whether the following GeoGebra commands were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | |
|--------------------------------|---|---|---|---|---|---|
| <input type="radio"/> Slope[] | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> Vertex[] | 5 | 4 | 3 | 2 | 1 | 0 |

4. What did you like about this GeoGebra workshop?

5. Is there anything you didn't like about this GeoGebra workshop? If yes, please explain!

Thanks for your participation!

Rating Survey for Workshop IV

NSF MSP Summer Institute Evaluation
Workshop 4: Functions and Export of Pictures

Code:

Mom	Dad	Year of Birth					

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith** Dad's name: **Dave Jones** YoB: **1976**
 Code: **MSDJ1976**







1. Please rate whether the GeoGebra workshop activities have been difficult or not.

(5 very difficult, 0 very easy)

- | | | | | | | | |
|-----------------------|-------------------------------------|---|---|---|---|---|---|
| <input type="radio"/> | Polynomial Functions | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Library of Functions | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Tangent to a Function | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Export of Pictures | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Inserting Pictures into a Word File | 5 | 4 | 3 | 2 | 1 | 0 |

2. Please rate whether the following GeoGebra tools were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | | |
|---|-----------------------|---|---|---|---|---|---|
|  | Intersect two objects | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Move | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Show / hide object | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Perpendicular line | 5 | 4 | 3 | 2 | 1 | 0 |
|  | New point | 5 | 4 | 3 | 2 | 1 | 0 |
|  | Tangent | 5 | 4 | 3 | 2 | 1 | 0 |

3. Please rate whether the following GeoGebra commands were difficult to use!

(5 very difficult, 0 very easy)

- | | | | | | | | |
|-----------------------|--------------|---|---|---|---|---|---|
| <input type="radio"/> | Root [] | 5 | 4 | 3 | 2 | 1 | 0 |
| <input type="radio"/> | Extremum [] | 5 | 4 | 3 | 2 | 1 | 0 |

4. What did you like about this GeoGebra workshop?

5. Is there anything you didn't like about this GeoGebra workshop? If yes, please explain!

Thanks for your participation!

Rating Survey for Home Exercise I

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: **Mary Smith**Dad's name: **Dave Jones**YoB: **1976**












Code: MSDJ1976

GeoGebra WS 1: Home Exercise "Equilateral Triangle"

Given a segment AB, construct an equilateral triangle ABC.

1. Did you know before how to solve this problem with paper, pencil, ruler and compass? yes no

2. Which of the following GeoGebra tools did you use? Please check!
If you checked "yes": Please rate if they were difficult to use!

	USED		RATING					
	yes	no	(5 very difficult, 0 very easy)					
 Segment through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Circle with center through point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Intersect two objects	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Polygon	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line bisector	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Show / hide object	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move drawing pad	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Zoom in / Zoom out	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Perpendicular line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
Others: _____			5	4	3	2	1	0

3. How long did you work on this exercise? _____ (minutes)

4. Please rate the difficulty of this home exercise. (5 very difficult, 0 very easy)
5 4 3 2 1 0

5. Which parts of the homework were difficult for you? Please, explain on the backside which difficulties you had.

Judith Preiner

Thanks for your participation!

Rating Survey for Home Exercise II

Code:

Mom		Dad		Year of Birth			

Code: Your parent’s initials (maiden name for your mother) and the year you were born.

Example: Mom’s maiden name: Mary Smith Dad’s name: Dave Jones YoB: 1976
Code: MSDJ1976

GeoGebra WS 2: Home Exercise “Symmetry Construction”

Look for a symmetric picture on the Internet and use it as a background image to make a symmetry construction on top of it. Tip: You can also reflect/rotate images – try it!

1. Which of the following GeoGebra tools did you use? Please check!

If you checked “yes”: Please rate if they were difficult to use!

	USED		RATING (5 very difficult, 0 very easy)
	yes	no	
Segment through two points	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Circle with center through point	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Intersect two objects	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Line through two points	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Move	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Polygon	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Line bisector	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Show / hide object	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Move drawing pad	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Zoom in / Zoom out	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Perpendicular line	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Parallel line	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Angle	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Mirror at line	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
New point	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Rotate around point	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Insert image	<input type="checkbox"/>	<input type="checkbox"/>	5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>
Others: _____			5 <input type="checkbox"/> 4 <input type="checkbox"/> 3 <input type="checkbox"/> 2 <input type="checkbox"/> 1 <input type="checkbox"/> 0 <input type="checkbox"/>

2. How long did you work on this exercise? _____ (minutes)

(5 very difficult, 0 very easy)

3. Please rate the difficulty of this home exercise. 5 4 3 2 1 0

4. Which parts of the homework were difficult for you? Please, explain on the backside which difficulties you had.

Judith Preiner

Thanks for your participation!

Rating Survey for Home Exercise III

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.

Example: Mom's maiden name: Mary Smith

Dad's name: Dave Jones

YoB: 1976





















Code: MSDJ1976

GeoGebra WS 3: Home Exercise "Quadratic Equation"

Create a dynamic construction for the quadratic equation $y = x^2 + p x + q$ using sliders for the parameters p , and q . Use the command `Vertex[]` for the quadratic equation and show the trace of this vertex. What happens to the trace if you move sliders p and q ?

1. Which of the following GeoGebra tools did you use? Please check!

If you checked "yes": Please rate if they were difficult to use!

	USED		RATING					
	yes	no	(5 very difficult, 0 very easy)					
 Segment through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Circle with center through point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Intersect two objects	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Polygon	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line bisector	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Show / hide object	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move drawing pad	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Zoom in / Zoom out	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Perpendicular line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Parallel line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Angle	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Mirror at line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 New point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Rotate around point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Insert image	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Slider	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Slope	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Text	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
Others: _____			5	4	3	2	1	0

2. Which of the following GeoGebra commands did you use? Please check!
If you checked “yes”: Please rate if they were difficult to use!

	USED		RATING					
	yes	no	(5 very difficult, 0 very easy)					
<input type="radio"/> Slope[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Vertex[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Others: _____			5	4	3	2	1	0

3. How long did you work on this exercise? _____ (minutes)
(5 very difficult, 0 very easy)
4. Please rate the difficulty of this home exercise. 5 4 3 2 1 0
5. Which parts of the homework were difficult for you? Please, explain which difficulties you had.

Rating Survey for Home Exercise IV

Code:

Mom		Dad		Year of Birth			

Code: Your parent's initials (maiden name for your mother) and the year you were born.























Example: Mom's maiden name: **Mary Smith**Dad's name: **Dave Jones**YoB: **1976**Code: **MSDJ1976**

GeoGebra WS 4: Home Exercise "Function Domino"

Create at least 4 cards of a "function domino" with GeoGebra and Word. Every card should have the graph of one function and the equation of another one on it. Choose different types of functions (e.g. linear and quadratic functions, absolute value function, trigonometric functions).

1. Which of the following GeoGebra tools did you use? Please check!

If you checked "yes": Please rate if they were difficult to use!

	USED		RATING					
	yes	no	(5 very difficult, 0 very easy)					
 Segment through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Circle with center through point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Intersect two objects	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line through two points	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Polygon	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Line bisector	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Show / hide object	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Move drawing pad	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Zoom in / Zoom out	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Perpendicular line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Parallel line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Angle	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Mirror at line	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 New point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Rotate around point	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Insert image	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Slider	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Slope	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Text	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Tangent	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
 Vector			5	4	3	2	1	0
Others: _____			5	4	3	2	1	0

Judith Preiner

Thanks for your participation!

2. Which of the following GeoGebra commands did you use? Please check!
If you checked “yes”: Please rate if they were difficult to use!

	USED		RATING					
	yes	no	(5 very difficult, 0 very easy)					
<input type="radio"/> Slope[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Vertex[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Root[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Extremum[]	<input type="checkbox"/>	<input type="checkbox"/>	5	4	3	2	1	0
<input type="radio"/> Others: _____			5	4	3	2	1	0

3. How long did you work on this exercise? _____ (minutes)
How many domino cards did you produce? _____ (cards)
- (5 very difficult, 0 very easy)
4. Please rate the difficulty of this home exercise. 5 4 3 2 1 0
5. Which parts of the homework were difficult for you? Please, explain which difficulties you had.

Appendix C

Coding Categories for Grounded Theory

This chapter lists the coding categories, keywords and their explanations that were developed using a Grounded Theory approach to analyze the answers to open ended questions as well as the helper reports. The results of the analysis are summarized in sections 5.2.2 (workshop feedback), 5.3.3 (home exercise feedback), and 7.4 (helper reports).

C.1 Introductory Workshops Feedback

Category I a: Workshop Design

- 1. Activities:** Teachers liked the active participation in hands-on activities and discussions offered in the workshops and wanted to extend these workshop components.
- 2. Support:** Teachers appreciated having plenty of helpers available for answering questions during the workshops and to help them follow along with the presenter's instructions.
- 3. Documentation:** Teachers asked for workshop handouts with detailed instructions in order to help slower participants to follow along during the workshops. They wanted to be able to review workshop contents at home and have these resources as guides for their home exercises.

Category I b: Workshop Contents

- 4. Contents:** Teachers enjoyed learning the basic use of the software and indicated that they were excited to learn more about it. Requests for specific content workshops for middle and high school level were made. Some teachers stated that too much information was presented in the workshops and that it was hard for them to memorize everything later on.

- 5. Examples:** Teachers liked the interactive dynamic figures presented in the workshops. Some teachers asked for more similar activities and examples for each topic.

Category I c: Workshop Activities

- 6. Constructions:** Teachers liked using technology for different constructions as well as learning how to actually create them. Participants enjoyed creating geometric constructions and the ease of use of GeoGebra. Some teachers complained that there were too many construction steps necessary in order to get the final product.
- 7. Algebra:** Teachers liked graphing lines, exploring slope, and graphically finding the solution of systems of equations with GeoGebra. Some also stated that these concepts would be easier to teach in this way.

Category I d: Workshop Implementation

- 8. Presentation:** Participants liked the presentation style of the instructor: thorough explanations, step-by-step instructions, and illustrative demonstrations.
- 9. Pace:** Participants gave feedback about the pace of the presentation.
- 10. Practice time:** Participants wanted to have more time to actively play with GeoGebra during the workshops and to work on the activities either alone or with a partner.
- 11. Popularity:** Participants stated that they enjoyed the introductory workshops.

C.2 GeoGebra Feedback

Category II a: Characteristics of GeoGebra

- 1. User friendly:** Participants characterized GeoGebra as user friendly, especially for teachers. They found it easy to use GeoGebra and reported that it makes drawing easier. Additionally they stated that the software is easily accessible.
- 2. Useful:** Participants found GeoGebra helpful and practical for teaching. They also liked the fact that GeoGebra is free and therefore available for both teachers and students.
- 3. Potential:** Participants liked the potential of GeoGebra for dealing with a variety of mathematical concepts on many difficulty levels.
- 4. Dynamic:** Teachers liked the fact that GeoGebra allows creation of dynamic figures for visualizing changes by moving objects with the mouse. They also liked the interactivity that allows their students to actively experiment with mathematical concepts.

- 5. Feedback:** Participants stated that they enjoyed using GeoGebra and that it is a remarkable tool.

Category II b: GeoGebra Tools and Features

- 6. Use of tools:** Teachers gave feedback about the use of different tools and their ease of use.
- 7. Helpful features:** Teachers found several features helpful for teaching (e.g. *Toolbar help*). They liked to change properties and they appreciated the fact that they could undo mistakes by using the *Undo* button.
- 8. Images & text:** Teachers liked that GeoGebra made it possible to insert images into a construction in order to use them to create more appealing activities for their students. They also liked inserting dynamic text into the graphics window.
- 9. Export:** Teachers liked the export possibilities for static pictures in order to create notes and worksheets for their students.

Category II c: Algebraic Input in GeoGebra

- 10. Algebraic input:** Teachers reacted positively to the fact that GeoGebra can be used to deliver concepts of algebra and that it allows the direct entry of equations and algebraic expressions. They especially liked finding solutions of systems of equations graphically.
- 11. Functions:** Teachers liked the function graphing capabilities of GeoGebra.

Category II d: Teaching with GeoGebra

- 12. Classroom use:** Participants stated that they planned to use GeoGebra in their classrooms, especially for topics that were usually difficult to understand for their students. Additionally, they found GeoGebra potentially useful for engaging their students' attention.
- 13. Methods:** Teachers found GeoGebra helpful for visualizing and exploring mathematical concepts. They characterized GeoGebra as a useful teaching tool that is easy to use and one that can help them present mathematics to their students in a new way.
- 14. Applications:** Teachers liked GeoGebra's flexibility and variety of application possibilities for teaching and learning mathematics.
- 15. Materials:** Teachers liked using GeoGebra in combination with other software (e.g. MS Word) in order to create teaching materials for their students.

C.3 Home Exercises Feedback

Category III a: Design of Home Exercises

1. **Feedback:** Teachers gave feedback on the difficulty level of the home exercises. They had fun playing with GeoGebra at home and characterized the software as easy to use and very intuitive.
2. **Content:** Teachers reported that it was helpful to know how to do the home exercises on paper before they worked on them using GeoGebra, but that it sometimes was challenging to remember all constructions steps necessary to finish the home exercises.

Category III b: GeoGebra Use at Home

3. **Use of tools:** Teachers reported difficulties with deciding which tools to use and remembering or finding out how to operate them.
4. **Algebraic input:** Teachers had difficulties using commands and typing in equations and functions.
5. **Features:** Teachers found inserting pictures difficult and couldn't remember how to insert text into the graphics window. Some teachers were confused about not being able to save the trace of an object. Teachers also gave feedback about the export of pictures from GeoGebra.

Category III c: General Computer Use

6. **Computer issues:** Teachers reported having problems with their computers at home (e.g. unable to save, unable to connect to the Internet).
7. **Pictures:** Teachers had problems with finding suitable pictures on the Internet that could be used within a GeoGebra construction. They also had difficulties resizing their images before inserting them into GeoGebra.
8. **MS Word use:** Teachers reported difficulties creating domino cards in MS Word using tables. They found it challenging to insert pictures, resize them within MS Word, and had troubles aligning images and equations on their cards.

C.4 Helper Reports

Category a: Mathematical Content

1. **Activities:** Reports about the construction process of dynamic figures and about the paper-and-pencil construction in workshop I.

Category b: Computer Issues

2. **Installation:** Reports about problems with the installation of GeoGebra and the Internet connection.
3. **Files:** Reports about saving and finding files on the notebook computers.
4. **Input device:** Reports about touchpad and mouse issues.
5. **MS Word:** Reports about general use of MS Word, as well as inserting pictures into an MS Word document.

Category c: GeoGebra in General

6. **Basic handling:** Reports about starting GeoGebra and opening GeoGebra files.
7. **GeoGebra concepts:** Reports about general concepts of GeoGebra (e.g. free and dependant objects).
8. **Selecting objects:** Reports about selecting one or several objects in GeoGebra.
9. **Other issues:** Reports about deleting objects, changing settings, and using keyboard shortcuts (e.g. *Esc*-key activates *Move* mode in GeoGebra).

Category d: GeoGebra Tools

10. **Toolbar:** Reports about dealing with the toolbar and activating tools.
11. **Use of tools:** Reports about the proper use of GeoGebra tools.
12. **Challenging tools:** Reports about challenging GeoGebra tools that required a lot of support from the helpers.

Category e: GeoGebra Features

13. **Menubar:** Reports about features that can be accessed over *Menubar*.
14. **Properties dialog:** Reports about using the *Properties dialog* in order to change properties of existing objects.
15. **Context menu:** Reports about features that can be accessed over the *Context menu*.
16. **Other features:** Reports about other GeoGebra features.

Category f: Algebraic Input and Commands

17. Input syntax: Reports about the algebraic input and its syntax in GeoGebra.

18. Use of commands: Reports about the use of commands in GeoGebra.

Appendix D

Complexity Analysis of DGS Tools

D.1 Cabri II Plus

In order to determine the general difficulty level of Cabri's dynamic geometry tools, they are characterized using the complexity criteria established in section 6.3.2:

Criteria 1: *The tool doesn't depend on already existing objects, or just requires existing points which can also be created 'on the fly' by clicking on the drawing pad. The order of actions is irrelevant and no additional keyboard input is required.*

Criteria 2: *The tool directly affects only one type of existing object or all existing objects at the same time and requires just one action. Again, the order of actions is irrelevant and no additional keyboard input is required.*

Criteria 3: *For this tool the order of actions is relevant, but no additional keyboard input is required.*

Criteria 4: *The tool requires already existing objects of the same type (except just points) or of different types. No additional keyboard input is necessary.*

Criteria 5: *The tool requires input into a dialog window and usually two or more actions whose order is relevant for a successful application.*

Criteria 1 and 2 specify dynamic geometry tools as 'easy to use' while criteria 3 and 4 define 'middle' tools. If a tool meets criteria 5 it is classified as a 'difficult to use' tool.

Cabri Tools that Correspond to Similar GeoGebra Tools

Dilate: This tool corresponds to the *Dilate object from point by factor* tool in GeoGebra, but doesn't require the input of a numerical dilation factor in order to be applied. Instead, after specifying an existing point as the center of dilation, another object can be dragged with the mouse in order to dilate it. Since two already existing objects

are required (e.g. point and circle), but no additional input is necessary, this tool meets complexity criteria 4 and therefore, could be considered a ‘middle’ tool.

Regular Polygon: This tool corresponds to the *Regular polygon* tool in GeoGebra, but doesn’t require any additional input in order to specify the number of vertices. While in GeoGebra the first and second click on the drawing pad specify two vertices of the regular polygon and open an input window to enter the total number of vertices, this tool works differently in Cabri. Here, the first click on the drawing pad specifies the center point, while the second click defines the first vertex of the regular polygon. The number of vertices can finally be determined by moving the pointer to the desired position of the second vertex, whereby the potential number of vertices is displayed next to the center point of the regular polygon. However, in Cabri this tool can be used in order to create convex regular polygons as well as star polygons, which is determined by either moving the pointer clockwise or counterclockwise when specifying the second vertex. Since for this tool the order of clicks is relevant, but no additional input is required, it meets complexity criteria 3 and can be considered to be part of the ‘middle’ tools group.

Compass: This tool corresponds to the *Circle with center and radius* tool in GeoGebra. The length of the radius needs to be specified first by either selecting an existing segment, two points, or a numerical value, before another click defines the center of the circle. Since the order of clicks is important and additional input is in most cases not necessary, this tool meets complexity criteria 3 and could be considered part of the ‘middle’ tools group.

Rotation: This tool corresponds to the *Rotate object around point by angle* tool in GeoGebra. The main difference is, that Cabri doesn’t provide a dialog window in order to specify the rotation angle whose size can be determined by either three points or an already existing number. Since additional input is not absolutely necessary for tool *Rotation*, but the order of clicks is relevant, it meets complexity criteria 3 and therefore could be considered a ‘middle’ tool.

Dilation: This tool corresponds to the *Dilate object from point by factor* tool in GeoGebra. Again, the dilation factor needs to be specified either using a number, or three points. Since additional input is not absolutely necessary for this tool, but the order of clicks is relevant, it meets complexity criteria 3 and thus, could be considered part of the ‘middle’ tools group.

Hide / Show Button: This tool corresponds to the *Checkbox to show and hide objects* tool in GeoGebra, but doesn’t require any additional input or dealing with a dialog window. Instead, a first click allows to create a button whose size can be determined by moving the mouse and clicking on the drawing pad again. Afterwards, an existing object can be selected which connects its visibility to the button created before. Since the order of clicks matters for this tool, but no additional input is required, it meets

complexity criteria 3. Therefore, it could be considered part of the ‘middle’ tools group.

Cabri Tools with Corresponding GeoGebra Features

Redefine Object: This tool corresponds to the *Redefine* feature in GeoGebra which can be accessed over the *Context menu*. Cabri allows a user to operate this tool without any additional input just by using the mouse. Clicking on an object opens a pop-up menu listing all available alternative definitions for the object. After selecting one of them (e.g. Point on Object), the required object(s) can be selected with the mouse and the definition of the initial object is modified. Since for this tool the order of clicks is relevant but no additional input is required, it meets complexity criteria 3 and could be considered part of the ‘middle’ tools group.

Initial Object(s): This tool represents the first of three steps allowing a user to create a customized tool or ‘Macro’ in Cabri. It corresponds to the ‘Input Objects’ tab of the *Create new tool* dialog window of GeoGebra. While in GeoGebra the desired objects can be selected from a list, Cabri doesn’t provide an extra dialog window for this tool. Instead, the initial objects need to be selected with the mouse before moving on to step two of the procedure (tool *Final Objects*). Although the complete process of creating a ‘Macro’ in Cabri is rather complex, the *Initial Objects* tool by itself could be considered a ‘middle’ tool because it requires already existing objects but no additional input, and therefore, it meets complexity criteria 4.

Final Object(s): This tool represents the second of three steps necessary to create a ‘Macro’ in Cabri. It corresponds to the ‘Output Objects’ tab of the *Create new tool* dialog window of GeoGebra. Again, Cabri allows a user to select the final objects of the ‘Macro’ in the dynamic figure without providing a dialog window. Subsequently, the tool *Define Macro...* needs to be activated in order to finish the creation process. Tool *Final Object(s)* requires already existing objects, but no additional input and therefore, meets complexity criteria 4, characterizing it as part of the ‘middle’ tools group.

Define Macro... : This tool is the last of three steps necessary to create a ‘Macro’ in Cabri. It corresponds to the ‘Name & Icon’ tab of the *Create new tool* dialog window in GeoGebra. Activating tool *Define Macro...* opens a dialog window where different text fields need to be filled in and an icon for the new tool can be created. Since this tool requires the prior application of tools *Initial Object(s)* and *Final Object(s)*, additional input into a dialog window, and the correct order of actions for its successful application, it meets complexity criteria 5 and could be considered a ‘difficult to use’ tool.

Equation or Coordinates: This tool allows display of the algebraic representation of geometric objects on the drawing pad by selecting them with the mouse. It corre-

sponds to the *Show Label: Value* feature on the ‘Basic’ tab of GeoGebra’s *Properties dialog*. Since this tool directly affects just one type of existing object and requires just one action, it meets complexity criteria 2 and thus, could be considered an ‘easy to use’ tool.

Fix / Free: This tool corresponds to the *Fix object* feature on the ‘Basic’ tab of GeoGebra’s *Properties dialog*. By selecting an object with the mouse, its position can be fixed making it impossible to move the object with the mouse. Applying the tool to the same object again restores its movability. Since this tool only affects one type of existing object and requires just one action without additional input, it meets complexity criteria 2 and therefore could be considered an ‘easy to use’ tool.

Trace On / Off: This tool corresponds to the *Trace on* feature of GeoGebra. Selecting an object with the mouse activates its trace in Cabri making the path of the object visible whenever it changes its position. Since this tool only affects one type of existing object and requires just one action without additional input, it meets complexity criteria 2 and therefore could be considered an ‘easy to use’ tool.

Color. . . : This tool corresponds to the ‘Color’ tab of GeoGebra’s *Properties dialog* and enables the user to change the color of objects. Activating the tool opens a pop-up window that allows a user to select the new color. By selecting an object afterwards, its color is changed accordingly. Since the color needs to be picked first, the order of actions is relevant for this tool. Therefore, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Fill. . . : This tool corresponds to the *Filling* feature on the ‘Style’ tab of GeoGebra’s *Properties dialog*. Activating the tool in Cabri opens a pop-up window, that allows a user to select the color of the filling. Afterwards, certain objects (e.g. triangle) can be selected which changes their filling according to the chosen color. Since the color needs to be selected first, the order of actions is relevant for this tool. Thus, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Text Color. . . : This tool corresponds to the ‘Color’ tab of GeoGebra’s *Properties dialog* and enables the user to change the color of an already existing text. Activating the tool opens a pop-up window that allows selection of the new color. Afterwards, this new color can be applied to a text by selecting it with the mouse. Since the color needs to be selected first, the order of actions is relevant for this tool. Thus, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Thick. . . : This tool corresponds to the *Line thickness* feature on the ‘Style’ tab of GeoGebra’s *Properties dialog*. Activating this tool in Cabri opens a pop-up window allowing selection of the desired thickness style which can be applied to an object afterwards. Since the thickness style needs to be selected first, the order of actions is relevant for this tool. Thus, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Dotted...: This tool corresponds to the *Line style* feature on the ‘Style’ tab of GeoGebra’s *Properties dialog*. Activating this tool in Cabri opens a pop-up window allowing the user to pick the desired formatting style which can be applied to an object by subsequently selecting it. Since the formatting style needs to be selected first, the order of actions is relevant for this tool. Thus, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Modify Appearance...: This tool summarizes the following features of GeoGebra: *Point style* feature in the *Options* menu, the ‘Decoration’ tab of the *Properties dialog*, and the ‘Axes’ tab of the *Properties dialog* for the drawing pad. After activating the tool in Cabri and picking the desired appearance, an object can be selected with the mouse in order to adapt its appearance to the selected style. Since the appearance style needs to be selected first, the order of actions is relevant for this tool. Thus, it meets complexity criteria 3 and could be considered a ‘middle’ tool.

Show Axes: This tool corresponds to the *Axes* feature which can be accessed over the *View* menu of GeoGebra. A single click on the drawing pad displays coordinate axes in Cabri. Since the tool doesn’t depend on already existing objects, it meets complexity criteria 1, and therefore, it could be considered an ‘easy to use’ tool.

Define Grid: This tool corresponds to the *Grid* feature which can be accessed over the *View* menu of GeoGebra. In Cabri, the tool requires an existing coordinate system to which it can be applied. Since it directly affects only the coordinate axes and requires just one action, this tool meets complexity criteria 2, and thus, it could be considered an ‘easy to use’ tool.

Cabri Tools Corresponding to Algebraic Input in GeoGebra

Vector Sum: Although this tool doesn’t exist in GeoGebra, its resulting sum of two vectors can be created using the input field. Selecting two already existing vectors in Cabri followed by a click on the drawing pad or an existing point, creates a new vector starting at the selected point and representing the sum of the two initial vectors. Since the tool requires two already existing vectors, it meets complexity criteria 4, and therefore, it could be considered part of the ‘middle’ tools group.

Parallel?: This tool corresponds to algebraic input including the symbol \parallel whose result is a Boolean variable in GeoGebra. In Cabri, selecting two objects (e.g. line and segment) creates a dynamic text stating whether or not these objects are parallel. The text can be positioned on the drawing pad by clicking one more time. Since the tool requires already existing objects of certain types, but no additional input is necessary, it meets complexity criteria 4 and could be considered a ‘middle’ tool.

Perpendicular?: This tool corresponds to algebraic input including the symbol \perp whose result is a Boolean variable in GeoGebra. In Cabri, selecting two objects (e.g. line

and segment) creates a dynamic text stating whether or not these objects are perpendicular. The text can then be positioned on the drawing pad by clicking one more time. Since the tool requires already existing objects of certain types, but no additional input is necessary, it meets complexity criteria 4 and could be considered a ‘middle’ tool.

Calculate. . . : Activating this tool opens a calculator dialog window which corresponds to GeoGebra’s input field. After typing in a calculation and clicking the ‘Equal’-button, the result can be dragged to the drawing pad, creating a text field at the selected position. Since this tool requires algebraic input into a dialog window, it meets complexity criteria 5 and therefore could be considered a ‘difficult to use’ tool.

Apply an Expression: This tool allows users to either evaluate an already existing expression for selected variable values, or application of an expression to the x -axis displaying the corresponding function graph. Both actions can be carried out using GeoGebra’s input field. However, in Cabri an expression has to be entered using tool *Expression* before clicking either on already existing numerical values for each variable used, or on the x -axis of an already displayed coordinate system. Since this tool requires additional input of an expression prior to the application, it meets complexity criteria 5 and could be considered a ‘difficult to use’ tool.

Numerical Edit: This tool corresponds to entering a number in GeoGebra using the input field. By clicking on Cabri’s drawing pad, a text field is opened allowing a user to enter a number either by clicking on the provided up and down arrows, or by using the keyboard. Since the keyboard input is not absolutely necessary for this tool, but the order of clicks is relevant, it meets complexity criteria 3 and therefore, it could be considered a ‘middle’ tool.

Expression: This tool corresponds to entering an algebraic expression using the input field in GeoGebra. A click on Cabri’s drawing pad opens a text field where an expression can be entered. Since this tool requires input into a text field, it meets complexity criteria 5 and therefore, it could be considered a ‘difficult to use’ tool.

Other Cabri Tools

Rotate and Dilate: This tool allows simultaneous rotation and dilation of an object about its centroid or another point. Dragging an object (e.g. a triangle) with the mouse applies the two transformations at the same time. If no other point is selected in advance, the centroid of the object serves as transformation center. Since the order of clicks is relevant in one of those cases, the tool meets complexity criteria 3, and therefore, it could be considered to be part of the ‘middle’ tools group.

Measurement Transfer: This tool transfers a measurement from an already existing number in counterclockwise direction to another object. After selecting a number,

clicking on an object (and sometimes also a point on the object) transfers the measurement and creates a new point on the object. Since the tool requires numerical input as well as several actions whose order is relevant, it meets complexity criteria 5, and therefore, it could be considered a ‘difficult to use’ tool.

Inverse: Selecting a circle and a point creates the image of the point under circle inversion. Since the order of clicks doesn’t matter but two objects are required, this tool meets complexity criteria 4 and thus could be considered part of the ‘middle’ tools group.

Collinear?: This tool creates a text stating whether three already existing points are collinear or not. Since this tool directly affects only one type of object (three points) and just requires selecting them in any order, it meets complexity criteria 2 and thus could be considered an ‘easy to use’ tool.

Equidistant?: This tool creates a text message stating whether or not a point is equidistant to two already existing other points. Since the order of clicks determines which two distances are compared, this tool meets complexity criteria 3, and therefore, it could be considered to be a ‘middle’ tool.

Tabulate: After clicking on the drawing pad, a table is displayed allowing the user to record numerical values from the dynamic figure. Since the numeric values need to be displayed before they can be inserted into the table, the order of actions is relevant for this tool. Therefore, it meets complexity criteria 3 and can be categorized as ‘middle’ tool.

Animation: Dragging and releasing an object causes it to move in the opposite direction according to the stretching level of the displayed spring. Since this tool directly affects only one type of existing object and requires just one action, it meets complexity criteria 2 and could be considered an ‘easy to use’ tool.

Multiple Animation. . . : Activating this tool opens a dialog window containing several buttons to create / remove necessary springs, as well as to play / stop / undo the animation. Animation springs can be created by clicking on an object and clicking on the drawing pad according to the desired stretching level and direction of the spring. Since this tool requires already existing objects as well as several actions whose order is relevant, it meets both complexity criteria 3 and 4 and could be considered part of the ‘middle’ tools group.

New Axes: This tool allows creation of a set of arbitrary coordinate axes by either clicking on three already existing points, or by clicking on empty spots on the drawing pad. The first click specifies the origin, while the second and third click determine the unit and direction of the y - and x -axis. Since the order of clicks is relevant for this tool, it meets complexity criteria 3, and therefore, it could be considered part of the ‘middle’ tools group.

D.2 Geometer's Sketchpad

In order to determine the general difficulty level of Geometer's Sketchpad's dynamic geometry tools and construction features, they are characterized using the complexity criteria established in section 6.3.2:

Criteria 1: *The tool doesn't depend on already existing objects, or just requires existing points which can also be created 'on the fly' by clicking on the drawing pad. The order of actions is irrelevant and no additional keyboard input is required.*

Criteria 2: *The tool directly affects only one type of existing object or all existing objects at the same time and requires just one action. Again, the order of actions is irrelevant and no additional keyboard input is required.*

Criteria 3: *For this tool the order of actions is relevant, but no additional keyboard input is required.*

Criteria 4: *The tool requires already existing objects of the same type (except just points) or of different types. No additional keyboard input is necessary.*

Criteria 5: *The tool requires input into a dialog window and usually two or more actions whose order is relevant for a successful application.*

Criteria 1 and 2 specify dynamic geometry tools as 'easy to use' while criteria 3 and 4 define 'middle' tools. If a tool meets criteria 5 it is classified as a 'difficult to use' tool.

Dynamic Geometry Tools

Selection Arrow Tool *Move*: This tool is equivalent to the *Move* tool in GeoGebra. Dragging an object with the mouse allows a change to its position on the drawing pad. Since this tool directly affects only one type of object at a time and requires just one action, it meets complexity criteria 2 and could be considered an 'easy to use' tool.

Selection Arrow Tool *Rotate*: This tool is equivalent to the *Rotate around point* tool in GeoGebra. After double clicking a point to specify it as the center of rotation, dragging an object rotates it about the center point. Since the order of actions is relevant for this tool, it meets complexity criteria 3 and could be considered part of the 'middle' tools group.

Selection Arrow Tool *Translate*: This tool is equivalent to the GeoGebra tool *Translate object by vector*. After double clicking a point to specify it as the reference point of the translation, dragging an object translates it with respect to the reference point. Since the order of actions is relevant for this tool, it meets complexity criteria 3 and could be assigned to the 'middle' tools group.

Point Tool: This tool is equivalent to the *New point* tool in GeoGebra and allows creation of new points 'on the fly' by clicking on the drawing pad. Since no existing objects are required, this tool meets complexity criteria 1 and therefore could be considered an 'easy to use' tool.

Compass Tool: This tool is equivalent to GeoGebra's *Circle with center through point* tool and allows creation of a circle by specifying the center before creating a point that lies on the circle. Although both points can be created 'on the fly', the order of clicks is relevant for this tool. Therefore, it meets complexity criteria 3 and could be considered part of the 'middle' tools group.

Straightedge Tool *Segment*: This tool is equivalent to the *Segment between two points* tool in GeoGebra and requires two points. These can either be selected or created 'on the fly' by clicking on the drawing pad. Since the order of clicks is not relevant for the creation of a segment between the points, this tool meets complexity criteria 1 and thus could be considered an 'easy to use' tool.

Straightedge Tool *Ray*: This tool is equivalent to GeoGebra's *Ray through two points* tool and requires two points which can also be created 'on the fly' by clicking on the drawing pad. The first point selected or created represents the starting point of the ray, whereas the second point determines its direction. Since the order of clicks is relevant for this tool, it meets complexity criteria 3 and could be considered a 'middle' tool.

Straightedge Tool *Line*: This tool is equivalent to the *Line through two points* tool in GeoGebra. Again, it requires two points which can be either selected or created 'on the fly'. The order of clicks is irrelevant for the application of this tool which meets complexity criteria 1 and could be considered an 'easy to use' tool.

Text Tool: This tool is equivalent to the *Insert text* tool in GeoGebra. After activating this tool, a text field can be created by clicking on the drawing pad and holding the mouse key in the depressed position while moving the pointer until the desired size is reached. This opens a formatting bar below the drawing pad and allows both the entry and formatting of the new text. Since input is necessary and more than one action is required, this tool meets complexity criteria 5 and could be considered a 'difficult to use' tool.

Custom Tool: This tool is equivalent to item 'Create new tool' in the *Tools* menu of GeoGebra. After creating a dynamic figure and selecting the desired input as well as output objects, activating this tool opens a dialog window allowing a user to both name the tool and open its script view. Since several actions are required in order to create a custom tool and a dialog window is involved, this tool meets complexity criteria 5 and thus could be considered a 'difficult to use' tool.

Construction Features

Construct Menu

Point On Object: This feature corresponds to GeoGebra's *New point* tool. After selecting a suitable object (e.g. circle, line), activating this feature creates a random point on the object. Since the feature directly affects just one type of object and requires only one action, it meets complexity criteria 2 and could be considered 'easy to use'.

Midpoint: This feature corresponds to the *Midpoint or center* tool in GeoGebra. After selecting a suitable object (e.g. segment), activating this feature creates the object's midpoint. Since only one type of object is directly affected, this feature meets complexity criteria 2 and therefore could be considered an 'easy to use' feature.

Intersection: This feature corresponds to GeoGebra's *Intersect two objects* tool. After selecting two intersecting objects (e.g. circle and line), their intersection point(s) are created. Since two already existing objects are required, this feature meets complexity criteria 4 and could be considered a 'middle' feature.

Segment: This feature corresponds to the *Segment between two points* tool in GeoGebra. After selecting two points, activating the feature creates a segment between them. Since only one type of existing object is required and the order of clicks is irrelevant, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Ray: This feature corresponds to GeoGebra's *Ray through two points* tool and requires two selected points. The order of clicks is relevant since the first point selected represents the starting point of the ray, whereas the second point determines its direction. Thus, the feature meets complexity criteria 3 and could be considered part of the 'middle' group.

Line: This feature corresponds to the *Line through two points* tool in GeoGebra. After selecting two points, activating the feature creates a line through them. Since only one type of existing object is required and the order of clicks is irrelevant, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Parallel Line: This feature corresponds to tool *Parallel line* in GeoGebra and requires the selection of a point as well as a straight object (e.g. segment). Since the feature involves different types of objects, it meets complexity criteria 4 and could be considered part of the 'middle' group.

Perpendicular Line: This feature corresponds to GeoGebra's *Perpendicular line* tool and requires the selection of a point as well as a straight object (e.g. segment). Since the feature involves different types of objects, it meets complexity criteria 4 and could be considered a 'middle' feature.

Angle Bisector: This feature corresponds to the *Angle bisector* tool in GeoGebra. It requires the prior selection of three points with the second point representing the vertex of the angle. Since the order of clicks is important, this feature meets complexity criteria 3 and thus could be considered a 'middle' feature.

Circle By Center + Point: The GeoGebra equivalent for this feature is the *Circle with center through point* tool. Two points need to be selected prior to activating the feature, whereby the first point selected determines the circle's center, and the other point specifies the radius. Since the order of clicks is relevant, this feature meets complexity criteria 3 and could be considered part of the 'middle' group.

Circle By Center + Radius: This feature corresponds to the *Circle with center and radius* tool in GeoGebra, but it doesn't require additional input to determine the radius. Instead, a point and a segment need to be selected in order to apply the feature successfully. The segment's length determines the circle's radius. Since different types of objects are required, this feature meets complexity criteria 4 and could be considered to be part of the 'middle' group.

Arc On Circle: This feature is similar to GeoGebra's *Circular arc with center through two points* tool, but requires the selection of a circle and two points laying on the circle. The order of selecting the two points is relevant for the length of the arc. This feature meets complexity criteria 3 as well as criteria 4 and could be considered a 'middle' feature.

Arc Through 3 Points: This feature corresponds to the *Circumcircular arc through three points* tool in GeoGebra. It requires the user to select three points with the order of clicks relevant to determine the starting and ending point of the arc. Therefore, this feature meets complexity criteria 3 and could be considered part of the 'middle' group.

Interior: This feature is similar to GeoGebra's *Polygon* tool and can be applied to a circle, arc, or group of more than two points. Although the feature only involves one type of object, the order of selecting the points is relevant for the shape of the resulting polygon. Therefore, this feature meets complexity criteria 3 and could be considered a 'middle' feature.

Locus: This feature corresponds to the *Locus* tool in GeoGebra and requires two selected objects. One of the objects needs to be dependant on the other one which has to be restricted to move along another object (e.g. circle). Since different types of objects are involved, this feature meets complexity criteria 4 and could be considered part of the 'middle' group.

Transform Menu

Mark Center: This feature specifies the selected point as center for future transformations. Since only one object is directly involved and just one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Mark Mirror: This feature specifies a selected straight object as mirror for future reflections. Since one object is directly involved and just one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Mark Angle: This feature defines a selected angle as default angle for future rotations. Either an angle measurement or three points can be used, although the order of selection is relevant for the points since the second point represents the vertex of the angle. Therefore, this feature meets complexity criteria 3 and could be considered part of the ‘middle’ group.

Mark Ratio: This feature requires the selection of two segments or three collinear points in order to calculate the ratio of their lengths or distances. Once defined, the ratio can be used as a dilation factor. The order of clicks determines the numerator and denominator of the ratio. Therefore, this feature meets complexity criteria 3 and could be considered a ‘middle’ feature.

Mark Vector: This feature requires two selected points in order to define a vector that can be used for future translations. The first point selected represents the starting point of the vector. Since the order of clicks is relevant for this feature, it meets complexity criteria 3 and could be considered to be part of the ‘middle’ group.

Mark Distance: This feature can be applied to a segment or distance measurement, and allows specification of a distance for future translations. Since just one object is directly involved and only one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Translate...: This feature is equivalent to the *Translate object by vector* tool in GeoGebra. Activating this feature opens a dialog window that allows the user to define parameters for the translation of the selected object(s). Since additional input is required, this feature meets complexity criteria 5 and could be considered ‘difficult to use’.

Rotate...: This feature is equivalent to the *Rotate object around point by angle* tool in GeoGebra. Activating this feature opens a dialog window that allows the definition of parameters for the rotation of the selected object(s). Since additional input is required, this feature meets complexity criteria 5 and could be considered ‘difficult to use’.

Dilate...: This feature is equivalent to the *Dilate object from point by factor* tool in GeoGebra. Activating this feature opens a dialog window that allows the definition

of parameters for the dilation of the selected object(s). Since additional input is required, this feature meets complexity criteria 5 and could be considered 'difficult to use'.

Reflect: This feature is equivalent to the *Mirror object at line* tool in GeoGebra. Activating this feature opens a dialog window that allows the definition of parameters for the reflection of the selected object(s). Since additional input is required, this feature meets complexity criteria 5 and could be considered 'difficult to use'.

Iterate. . . : This feature is equivalent to the *Iterate* command in GeoGebra, which can be applied using the input field. After selecting a suitable object, activating this feature opens a dialog window in which parameters for the iteration can be specified. Since additional input is required, this feature meets complexity criteria 5 and could be considered 'difficult to use'.

Measure Menu

Length: This feature corresponds to the *Distance and length* tool of GeoGebra. It can be used to measure the length of a selected segment and to display it as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Distance: This feature corresponds to the *Distance and length* tool of GeoGebra and can be used to measure the distance between two selected objects (e.g. point and segment). Since different types of objects are involved, this feature meets complexity criteria 4 and could be considered to be part of the 'middle' group.

Perimeter: This feature corresponds to the *Distance and length* tool of GeoGebra and can be used to measure the perimeter of a polygon that was defined by using feature *Interior*. The perimeter is displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Circumference: This feature corresponds to the *Distance and length* tool of GeoGebra and can be used to measure the circumference of a selected circle. The circumference is displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Angle: This feature is equivalent to the *Angle* tool of GeoGebra and can be used to measure the angle between three selected points. The angle measure is displayed as a dynamic text. The order of clicks is relevant for the specification of the angle's vertex which is identified as the second point selected. Therefore, this feature meets complexity criteria 3 and could be considered a 'middle' feature.

Area: This feature is equivalent to the *Area* tool of GeoGebra and can be used to measure the area of a selected object (e.g. circle, polygon) and display it as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Arc Angle: This feature is equivalent to the *Angle* tool of GeoGebra and can be used to measure the length of an arc between two or three selected points that lie on a circle. The arc length is displayed as a dynamic text. Since the circle as well as the points need to be selected, this feature involves different types of objects and therefore meets complexity criteria 4. Thus, this feature could be considered part of the ‘middle’ group.

Arc Length: This feature corresponds to the *Distance and length* tool of GeoGebra and can be used to measure the length of a selected arc and display it as a dynamic text. The arc could also be specified by two or three points that lie on a circle which needs to be selected together with the points. Since in this case different types of objects are involved, this feature meets complexity criteria 4 and therefore could be considered to be a ‘middle’ feature.

Radius: This feature corresponds to the *Distance and length* tool of GeoGebra and can be used to measure the radius of a selected object (e.g. circle) which is displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Ratio: This feature is equivalent to *algebraic input* in GeoGebra and measures the ratio defined by two segments or three collinear points. The ratio is displayed as a dynamic text. The order of clicks is relevant since the order determines the numerator and denominator of the ratio. As two or more already existing objects are required, and the order of clicks is important, this feature meets both complexity criteria 3 and criteria 4 and could be considered a ‘middle’ feature.

Calculate: This feature is equivalent to *algebraic input* in GeoGebra. Activating this feature opens a calculator window. Since input is required for this feature, it meets complexity criteria 5 and could be considered ‘difficult to use’.

Coordinates: This feature corresponds to GeoGebra’s *algebra window* and can be used to determine the coordinates of the selected point and display them as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered an ‘easy to use’ feature.

Abscissa (x): This feature is equivalent to *algebraic input* in GeoGebra and determines the x -coordinate of the selected point which is displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered ‘easy to use’.

Ordinate (y): This feature is equivalent to *algebraic input* in GeoGebra and determines the y -coordinate of the selected point which is displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Coordinate Distance: This feature is equivalent to *algebraic input* in GeoGebra and can be used to measure the distance between two selected points within a coordinate system which is displayed as a dynamic text. Since only one type of objects is involved and the order of clicks is irrelevant, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Slope: This feature corresponds to the *Slope* tool of GeoGebra and can be used to measure the slope of a selected straight object and display it as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered an 'easy to use' feature.

Equation: This feature corresponds to the *algebra window* of GeoGebra and allows the user to determine the equation of a selected object (e.g. circle) which is then displayed as a dynamic text. Since just one object is involved and only one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Graph Menu

Define Origin: This feature defines the selected point as origin of a new coordinate system. Since only one object is involved and just one action is required, this feature meets complexity criteria 2 and could be considered 'easy to use'.

Mark Coordinate System: This feature corresponds to the 'Axes' feature in the *View* menu of GeoGebra. It can be used to mark the actual coordinate system allowing it to be used in future plots and coordinate measurements. Since no objects are directly involved, this feature meets complexity criteria 1 and could be considered an 'easy to use' feature.

Grid Form (Polar, Square, Rectangular): This feature allows display of one out of three grid types within a coordinate system. Since no objects are directly involved, this feature meets complexity criteria 1 and could be considered 'easy to use'.

Show / Hide Grid: This feature corresponds to the 'Grid' feature in the *View* menu of GeoGebra and allows users to show or hide the coordinate grid. Since no objects are directly involved, this feature meets complexity criteria 1 and could be considered 'easy to use'.

Snap Points: This feature corresponds to the 'Point capturing' feature in the *Options* menu of GeoGebra. This feature makes it easier to create points with integer coordinates. Since no objects are directly involved, this feature meets complexity criteria 1 and could be considered an 'easy to use' feature.

Plot Points...: This feature corresponds to *algebraic input* in GeoGebra. Activating this feature opens a dialog window which allows users to enter coordinates of a new point. Since input is required, this feature meets complexity criteria 5 and could be considered ‘difficult to use’.

New Parameter...: This feature corresponds to *algebraic input* in GeoGebra. Activating this feature opens a dialog window which allows users to enter the name and value of a new parameter as well as to select its unit. Since input is required for this feature, it meets complexity criteria 5 and could be considered ‘difficult to use’.

New Function...: This feature corresponds to *algebraic input* in GeoGebra. Activating this feature opens a dialog window which allows users to enter a new function and display it as a dynamic text. Since input is required for this feature, it meets complexity criteria 5 and could be considered ‘difficult to use’.

Plot New Function...: This feature corresponds to *algebraic input* in GeoGebra. Activating this feature opens a dialog window allowing entry of a new function which is displayed as dynamic text and plotted within a coordinate system. Since input is required for this feature, it meets complexity criteria 5 and could be considered ‘difficult to use’.

Derivative: This feature corresponds to the command *Derivative* in GeoGebra and can be used to display the derivative of a selected function as dynamic text. Since only one object is involved and one action is required, this feature meets complexity criteria 2 and therefore could be considered ‘easy to use’.

Tabulate: Activating this feature creates a table using the selected measurement(s). Since several objects of the same type are usually involved, this feature meets complexity criteria 4 and could be considered part of the ‘middle’ group.

Add Table Data...: After selecting an already existing table, activating this feature opens a dialog window allowing a user to specify how many rows should be added to the table. Afterwards, the object, whose measurement was initially used to create the table, needs to be moved in order to successively record new measurements. Since a dialog window as well as several actions are involved when applying this feature, it meets complexity criteria 5 and could be considered a ‘difficult to use’ feature.

Remove Table Data...: After selecting an already existing table with more than one row, activating this feature opens a dialog window that allows a user to specify which table entries should be removed. Therefore, this feature meets complexity criteria 5 and could be considered ‘difficult to use’.